

# Nonlinear Regression Model for Ride on Railway

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## Summary

The portable diagnosis system – SPD – evaluates the safety and ride quality aspects of the railway vehicles and the technical condition of the rail-vehicle interface. The objective of this article is to estimate the nonlinear regression model associated with the ride quality or motion behavior, by applying fuzzy clustering algorithms to the geometric data obtained from the technical condition of the railway-vehicle interface and measuring quasi-static lateral acceleration  $y_{qst}^*$  in different vehicles. The performance will be evaluated by comparing the measured acceleration  $y_{qst}^*$  with the acceleration calculated in our model  $y_{qstM}^*$  for 15 different vehicles. The obtained results will be then compared with the results of the multiple linear regression model used previously for the same purpose.

**Keywords:** rail vehicle, heat release rate, fire, EU499 Eureka Project, transfeu Project, Metro Project, CFD

## 1. Introduction

The ride quality of passenger railway vehicles, according to the UIC-518 norm from the International Union of Railways, is connected with the value of acceleration  $y_{qst}^*$  which should have a limit value of 1,5 m/s<sup>2</sup> [12]. Due to the high cost involved in measuring of the acceleration  $y_{qst}^*$  of each vehicle, it is necessary to obtain a tool (model) that allows predicting the behavior of the acceleration  $y_{qst}^*$  according to the measured 23 geometric variables which are routinely measured in the normal preventive maintenance routine of the railway, without the need of performing a  $y_{qst}^*$  measuring process. Many of the traditional method used to solve this problem are based on global models like Polynomials (ARMA, ARX, NARMX, NARMAX) [6, 17, 20], radial basis functions and neural network [21, 10, 16, 19], fuzzy clustering [1, 17] among others some of them are used in similar railway applications in the world [7, 8].

The fuzzy clustering is to approximate a nonlinear regression problem by decomposing into several local linear models; this approach has advantages in comparison to global nonlinear models [1, 24]. The model structure is easy to understand and interpret, both qualitatively and quantitatively. Besides, the approach has computational advantages and goes down to straightforward adaptive and learning algorithms.

To show the feasibility of the approach, we will compare the obtained results using fuzzy clustering with the Babuska toolbox [1] with the results obtained with the multiple linear regression model used previously for the same purpose [24].

This article is part of the development of SPD (Portable Diagnostic System, [3–5, 13, 18, 23–24], which consists of the measurement of the vehicle's variables allowing the identification of the technical condition for the vehicle-railway interface.

Section 2 of paper introduces the element for the regression used in the SPD system; in section 3 we will review the nonlinear regression; section 4 will detail the fuzzy clustering methodology; and sections 5 and 6 will show the results the comparison with NRL (multiple nonlinear regression) [24] and conclusions respectively.

## 2. Study system

The Metro system of Medellín was created on may 1979 by the Municipality and Antioquia Department, allowing the creation of the Metro Company. Description of the railroad (Fig.1):

- **Line A:** paralell to the Medellín River and with the length of 23.2 km, with 19 stations in North to South direction.

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- **Line B:** it starts from the centre of the city in San Antonio B Station and goes westwards. It has the length of nearly 5.6 km and has 7 stations.
- **Linking Line:** it connects the two lines described above and has the length of 3.2 km.
- **Line K:** it is a cable transport system that connects the Acevedo Station. It consists of 4 stations the length of 2.4 km.



Fig. 1. Metro System of Medellín

In order to extract the data, both estimators given by the UIC 518 standard and the geometric variables, the complete railroad of the train is taken and the measuring points were classified by sections, just as the standard UIC-518 recommends. The three zones proposed by the standard are considered: tangent tracks, large radius curve tracks and low radius curve tracks; however, the lengths of sections composing different zones were adapted according to the distribution of the Line A road of the Metro system. The considered lengths were:

- tangent track: 160 m,
- large radius curve track: 70 m,
- short radius curve track: 70 m.

## 2.1. Data acquisition

The Portable Diagnostic System – SPD – is a unique solution for railway systems which, apart from evaluating safety, ride quality and monitoring the condition of geometric parameters of the track-vehicle interface, also allows carrying out the multidimensional monitoring of the condition and to determine the failures of passenger vehicles of the Metro [3–5, 24].

To develop this diagnosis tool, different methodologies were used, grouping several modern and effective methods in diagnosis tasks, which go from the selection of measurement points, through the method of evaluation of compliance with UIC-518 standard until the utilization of an optimized forecast method [3–5, 12, 24].

The system is composed of eight modules: sensors, signal processing, condition monitoring, condition testing, incipient failures detection in the wheel-rail

interface, decisions support, forecast and presentation. In the Fig. 2, the SPD module structure is shown.

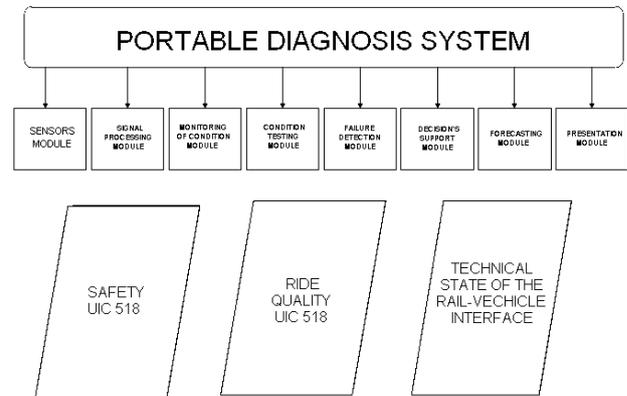


Fig. 2. Module of the SPD

The signal obtained by the SPD allows calculating the lateral and longitudinal forces generated in the wheel-rail along the track which are necessary for safety evaluation. The UIC-518 standard describes the experimental procedures to follow in order to carry out the motion tests and the analysis of the results, in terms of quality and rolling from the point of view of dynamic behavior in relation to safety, railroad wear and running behavior (ride quality) with the purpose of an approval for the international railway traffic. Table 1 presents the different estimators considered by the standard. It was necessary to acquire acceleration and forces signals in different parts of the train to calculate the estimators [19].

Since this article is limited to the ride quality evaluation, the estimator to use will be the acceleration  $y_{qst}$ . According to the UIC-518 standard [12] the limit value of this acceleration of  $1,5 \text{ m/s}^2$ , defines the ride quality or motion behavior of the vehicle.

This estimator is obtained from the lateral acceleration signal, taken from the vehicle body. These measurements are filtered by Butterworth 8<sup>th</sup> digital filter, order 8 and cut-off frequency of 20 Hz.

## 2.2. Geometric variables

Among the current maintenance routines of the railway system, different geometric variables that give an idea of the technical condition of the rail. Table 2 contains information also on equivalent conicity which is related to vehicles.

## 3. Principles of regression

Generally, fuzzy systems are approximations of functions. Because of this, they can also be used in nonlinear regression problems. The nonlinear regres-

Table 1

## Estimators for safety, ride quality, and track fatigue according to the UIC-518 Standard

Estimator	Description	Units	Limit Value
SY2m	Sum of guiding forces for axle	kN	66.7
SY2m (99,85%)	Sum of guiding forces for axle, Percentile 99.85%.	kN	66.7
SY2m (0,15%)	Sum of guiding forces for axle, Percentile 0.15%.	kN	66.7
sSY	Weighted r.m.s of Sum of guiding forces por axle	kN	33.3
Yqst	Quasi-static force between wheel and rail	m/s <sup>2</sup>	60
y:*q	Lateral acceleration in the vehicle body.	m/s <sup>2</sup>	2.5
y:*q (99,85%)	Lateral acceleration in the vehicle body, Percentile 99.85%	m/s <sup>2</sup>	2.5
y:*q (0,15%)	Lateral acceleration in the vehicle body, Percentile 0.15%	m/s <sup>2</sup>	2.5
sy:*q	Weighted r.m.s of Lateral acceleration in the vehicle body	m/s <sup>2</sup>	0.5
y:*qst	Quasi-static acceleration in the vehicle body	m/s <sup>2</sup>	1.5
z:*q	Vertical acceleration in the vehicle body	m/s <sup>2</sup>	2.5
z:*q (99,85%)	Vertical acceleration in the vehicle body, Percentile 99.85%	m/s <sup>2</sup>	2.5
z:*q (0,15%)	Vertical acceleration in the vehicle body, Percentile 0.15%	m/s <sup>2</sup>	2.5
sz:*q	Weighted r.m.s of Vertical acceleration in the vehicle body	m/s <sup>2</sup>	0.75
I	Cant deficiency	mm	150

Table 2

## State of variables

Geometric Variable	Description	Units	Limit Value
X1	Equivalent conicity with standard deviation of 1.25 under the UK method	N/A	
X2	Equivalent conicity with standard deviation of 2.5 under the UK method	N/A	
X3	Equivalent conicity with standard deviation of 3.75 under the UK method	N/A	
X4	Maximum speed vehicle	km/h	80
X5	Standard deviation of the vertical alignment	Mm	2.3
X6	Standard deviation of the horizontal alignment	Mm	1.5
X7	Cant deficiency	Mm	150
X8	Curve radius	M	0
X9	Horizontal alignments	Mm	3
X10	Height difference between the head of the high and low thread	Mm	3
X11	Vertical alignments	Mm	10
X12	Gap between the internal rail faces	Mm	3
X13	Synthetic coefficient of the railroad quality	Mm	0
X14	Vertical wear of the head rail for the high rail (east-south)	Mm	12
X15	Vertical wear of the head rail for the high rail (west-north)	Mm	12
X16	r.m.s of the corrugation for the high rail for a wave length between 30 and 100 mm	Mm	10
X17	Excess percentage for the high rail for a wave length between 30 and 100 mm	%	50
X18	r.m.s of the corrugation for the high rail for a wave length between 100 and 300 mm	Mm	20
X19	Excess percentage for the high rail for a wave length between 100 and 300 mm.	%	50
X20	r.m.s of the corrugation for the low rail wave length between 30 and 100 mm	Mm	10
X21	Excess percentage for the low rail for a wave length between 30 and 100 mm	%	50
X23	r.m.s of the corrugation for the low rail for a wave length between 100 and 300 mm	Mm	20
X24	Excess percentage for the low rail for a wave length between 100 and 300 mm	%	50

sion is a modeling of static dependence of the response of a variable called regressor, where:  $y \in Y \subset \mathfrak{R}$ , is a regression vector,  $x = [x_1, x_2, \dots, x_p]^T$ , over the  $X \subset \mathfrak{R}^p$  domain. The elements of the regression vector can be called regressors and the  $X$  domain can be called regressor space. The system generated by the data can be described by:

$$y \approx f(x) \quad (1)$$

The deterministic function  $f(\cdot)$  captures the dependence of  $y$  in  $x$ , and the symbol  $\approx$  reflects the characteristics of  $y$  that are not exact in function of  $x$ . The objective of the regression is to use the data in order to build a function  $F(x)$  as an approximation to  $f(x)$  not only because of the data, but because of the domain itself. The definition of a reasonable approximation depends on the purpose for which the model is built. If the objective of the model is to obtain predictions of  $y$ , the accuracy must be the most relevant criteria. The accuracy insufficiency is usually known as the integral error over the domain.

$$I = \int_x \|f(x) - F(x)\| dx \quad (2)$$

Generally, this error can not be computed, since the value of  $f$  is only known with the availability of the data. However, the mean of the error prediction of the available data is often used

$$J = \frac{1}{N} \sum_{i=1}^N \|f(x_i) - F(x_i)\| \quad (3)$$

where  $N$  is the number of data in the sample.

Apart from the prediction accuracy, the objective can also be to obtain a model which can be used in order to analyze and understand the real properties of the data generator system. The potential of fuzzy models is that they describe systems as the collection of simple local sub-models expressed by rules. The rules can be formulated using a natural language which is more understandable than a mathematical language. The rules can also be combinations of analytical models commonly used in the control field of engineering, like the local linear models in Takagi-Sugeno [20].

The input of our model are 23 geometric variables of the rail state, and with them, the modeled acceleration  $Y_{qstM}$  is calculated. An arrangement is conformed having a row for each of the  $n$  geometric variables measured for each section, and a column for each of the  $N$  sections. This arrangement is called the matrix of observation  $X$ .

$$Z = \begin{bmatrix} z_{11} & z_{12} & \cdots & z_{1N} \\ z_{21} & z_{22} & \cdots & z_{2N} \\ \vdots & \vdots & \vdots & \vdots \\ z_{n1} & z_{n2} & \cdots & z_{nN} \end{bmatrix} \quad (4)$$

Traditionally, the clustering terminology defines the columns of the matrix of observation  $X$  as characteristics or attributes, while the rows are called patterns or objects.

#### 4. Fuzzy clustering logics

It is defined as *cluster*, the subset of data which are more similar among them than with other data from another subset. There are different types of data association or clustering, one of the most popular the *Hard clustering* which refers to grouping data in specific clusters mutually exclusive (see Fig. 3), meaning that the data belongs only to one cluster and not to several clusters at the same time. In Figure 3, the data  $z_5$  could belong to both clusters  $c_1$  and  $c_2$ , this data is not taken into account when using the Hard cluster.

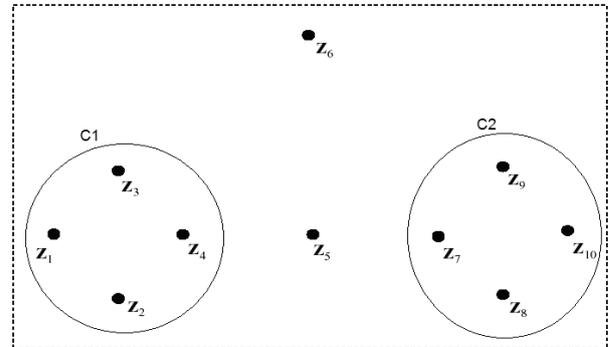


Fig. 3. Data set [1]

It is reasonable to think that on the border of two clusters  $c_1$  and  $c_2$  there are some points which have a degree of belonging to both clusters. The algorithm *c-means* allows that each point belongs to a cluster with a certain degree of belonging, so each point belongs to several clusters. This makes the fuzzy clustering, in some real situations, to be more natural than the Hard clustering.

##### 4.1. Partition Fuzzy

The objective of clustering is to divide the data set  $Z = \{z_1, z_2, \dots, z_N\}$  in  $c$  clusters ( $2 \leq c \leq N$ ), that partition  $U = [u_{ik}]$ , where  $u_{ik}$  is the degree of belonging of  $i$ -th point to the cluster  $k$ .  $U$  represents a fuzzy partition if

the points meet the following conditions:

$$u_{ik} \in [0,1] \quad 1 \leq i \leq c, \quad 1 \leq k \leq N, \quad (5)$$

$$\sum_{i=1}^c u_{ik} = 1 \quad 1 \leq k \leq N, \quad (6)$$

$$0 < \sum_{k=1}^N u_{ik} < N \quad 1 \leq i \leq c. \quad (7)$$

Defining the fuzzy partition space as:

$$M_{fc} = \left\{ U \in \mathfrak{R}^{c \times N} \left| \begin{array}{l} u_{ik} \in [0,1], \forall i, k; \sum_{i=1}^c u_{ik} = 1, \forall k; \\ \forall k; 0 < \sum_{k=1}^N u_{ik} < N, \forall i \end{array} \right. \right\}.$$

## 4.2. Algorithm for Fuzzy C-Means

There are different algorithms for fuzzy clustering, the most used is the „C-Mean” algorithm. This algorithm makes the data partition, and it can be minimized the objective function [17]:

$$J(Z; U, V) = \sum_{i=1}^c \sum_{k=1}^N u_{ik}^m d_{ik}^2 \quad (8)$$

where:

$Z = \{z_1, z_2, \dots, z_N\}$  – is the data set to classify; (9)

$U = [u_{ik}] \in M_{fc}$  – is the partition matrix  $Z$ ; (10)

$V = \{v_1, v_2, \dots, v_c\}$ ,  $v_i \in \mathfrak{R}^n$  – is the centre vector (clusters) to find; (11)

$d_{ik}^2 = \|z_i - v_i\|^2$  – is the Euclidian norm, distance from the data to the center of the cluster; (12)

$m \in [1, \infty)$  – is an exponent that determines the fuzziness of the obtained cluster; (13)

The steps of the algorithm are:

- to select a belonging matrix,
- to start the number of clusters,
- to calculate the centroid of the clusters:

$$v_i = \frac{\sum_{k=1}^N u_{i,k}^m z_k}{\sum_{k=1}^N u_{i,k}^m} \quad (14)$$

- to calculate the Euclidean distance:

$$d_{ik} = (z_k - v_i)^T (z_k - v_i) \quad (15)$$

- to update the belonging matrix:

$$u_{ik} = \frac{1}{\sum_{j=1}^c \left( \frac{d_{ik}}{d_{jk}} \right)^{\frac{1}{m-1}}} \quad (16)$$

The equation (14) gives the value  $v_i$  which is the weighted average of the data belonging to a cluster, where the weights are the belonging functions. This algorithm presents the following disadvantages:

- the final results depend on the final partition,
- the number of clusters is defined at the beginning of the algorithm,
- the Euclidean distance method allows detecting only spherical clusters.

This very last feature is a drawback because the ideal shape of data grouping is given by an ellipse (Fig. 4), so the most appropriate algorithm is the one called „Gustafson-Kessel” because this one looks for hyper ellipsoids clusters, which detects the quasi-linear behavior of data very well.

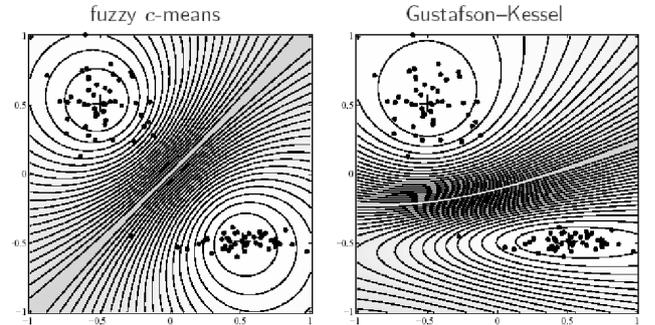


Fig. 4. Clusters of different shape [1]

## 4.3. Gustafson-Kessel (GK) algorithm

This algorithm is found among the adaptive distance algorithms. This one extends the fuzzy c-means by choosing a different norm  $B_i$  for each cluster instead of keeping it constant.

$$d_{ikB}^2 = (z_k - v_i)^T B_i (z_k - v_i) \quad (17)$$

where:  $B_i$  are the possible optimization matrixes of the objective function, and correspond to the covariance of each cluster.

Then, the objective function is defined as follows:

$$J(Z; U, V) = \sum_{i=1}^c \sum_{k=1}^k u_{ik} d_{ikB_i}^2 \quad (18)$$

In order to obtain a viable solution,  $B_i$  must be somehow limited. In this case, we will keep its volume constant by fixing the determinant of  $B_i$ :

$$B_i = [\rho_i \det(F_i)]^{\frac{1}{2}} F_i^{-1} \quad (19)$$

where:  $F_i$  is the covariance matrix for each cluster.

The GK algorithm fits the purpose of identification because it has the following characteristics:

- the cluster dimension becomes limited by measuring the distance and by the definition of the clusters prototype as a point;
- in comparison to other algorithms, GK is relatively insensitive to the initialization of the partition matrix.

Once we have the groups of data, the next step is to derive the interference rules which identify a fuzzy model. To achieve that, there are different types like:

- Mamdani: fuzzy rules with fuzzy antecedents and fuzzy consequents.
- Takagi-Sugeno (TS): fuzzy rules with fuzzy antecedents and consequents that could be expressed in a simple way like the first order linear model [20].

Because the TS fuzzy model is an effective tool for the approximation of nonlinear systems based on the information of inputs and outputs through the interpolation of local linear models, which for this case are determined by the cluster, we use this TS model in the solution of the identification of the model we are looking for. The solution consists of projecting the belonging of the obtained cluster in the desired space (Fig. 5), thus obtaining belonging functions from the cluster.

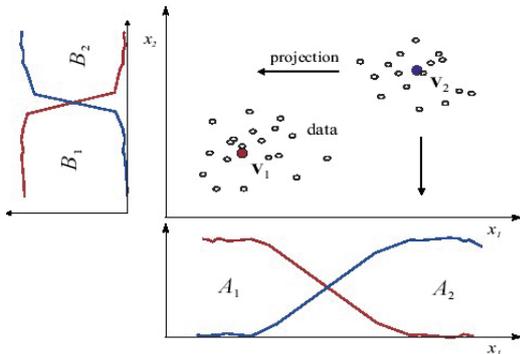


Fig. 5. Extraction of rules by fuzzy clustering [1]

#### 4.4. Takagi-Sugeno Model

In the Takagi-Sugeno model, the consequent rules are function of the inputs [20]:

$$R_i: \text{If } x \text{ is } A_i \text{ Then } y_i = f_i(x), \quad i = 1, 2, \dots, K \quad (20)$$

where:  $x \in \mathfrak{X}$  is the input variable (antecedent),  $A_i$  is a multidimensional fuzzy set (cluster),  $y_i$  is the output variable (consequent),  $R_i$  is the its rule and  $K$  is the number of rules of the rules set.

The consequent function can be linearly expressed as:

$$y_i = a_i^T x + b_i \quad (21)$$

Substituting (21) in (20) we get:

$$R_i: \text{If } x \text{ is } A_i \text{ then } y_i = a_i^T x + b_i \quad (22)$$

Given the outputs of the individual consequents  $y_i$ , the global output and the Takagi-Sugeno model is calculated by:

$$y = \frac{\sum_{i=1}^K \beta_i(x) y_i}{\sum_{i=1}^K \beta_i(x)} \quad (23)$$

where:  $\beta_i$  is the commitment degree of the antecedent of the its rule, calculated as the belonging degree of  $x$  in the interior of the  $A_i$  cluster:

$$\beta_i(x) = \mu_i(x) \quad (24)$$

normalizing,

$$h_i(x) = \frac{w_i(x)}{\sum_j w_j(x)} \quad (25)$$

therefore the TS model could be interpreted as a quasi-linear model with dependence on the input  $x$  parameter.

$$y = \sum_{i=1}^r h_i(x) \cdot (a_i^T \cdot x + b_i) \quad (26)$$

Fig. 6 and 7 shows an example of a function  $y = f(x)$ , represented by four TS rules.

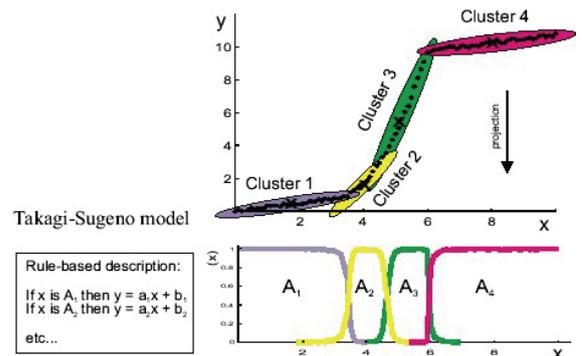


Fig. 6. Takagi-Sugeno fuzzy clustering [20]

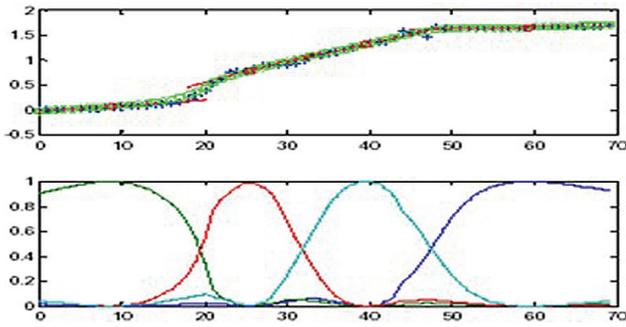


Fig. 7. GK y TS fuzzy clustering [1]

The antecedent of each rule defines a valid zone (fuzzy) for the correspondent linear model of the consequent. The global output function is calculated through weighting the local linear models.

### 5. Numerical result

During this work, the toolbox developed by Babuska [1] from the Delft Centre for System and Control was used. This tool was developed to be worked on MATLAB (Fig. 8).

$$\begin{matrix}
 & & & & \text{término general} \\
 & z_{11} & \dots & z_{1j} & \dots & z_{1K} \\
 & \vdots & & \vdots & & \vdots \\
 z_{i1} = \frac{x_{i1} - \bar{x}_1}{s_1} & \dots & z_{ij} = \frac{x_{ij} - \bar{x}_j}{s_j} & \dots & z_{iK} = \frac{x_{iK} - \bar{x}_K}{s_K} \\
 & \vdots & & \vdots & & \vdots \\
 z_{n1} & \dots & z_{nj} & \dots & z_{nK}
 \end{matrix}$$

Fig. 8. Normalization of a matrix

where:  $x$  – is the measure of each variable,  $S$  – is the standard deviation.

The quality of the model is evaluated by calculating the average error, its equivalent in the used toolbox corresponds to the percentile variance accounted (VAF) [1], between the real and the estimated data. This coefficient is obtained between two signals:

$$VAF = 100\% \cdot \left[ 1 - \frac{\text{var}(y1 - y2)}{\text{var}(y1)} \right] \quad (27)$$

where the value of VAF will be 100% if both signals are equal. If the values are quite different, the value of VAF will tend to zero.

For each vehicle, the following procedure was followed:

- the matrix is normalized;

- the matrix is divided in two sections, one for the identification of the model and the other to carry out the verification of the obtained model,
- the real acceleration with the obtained model by multiple linear regression was plotted [22, 14]. This was also used to determine the ride quality model [24], therefore it will be a reference to validate our results and the obtained result by the fuzzy clustering method,
- the VAF coefficient is calculated to determine the accuracy of the model compared with the real data.

The best results were obtained using the toolbox with the following parameters:

- FM.c = 12; % number of clusters,
- FM.m = 2.8 % fuzziness parameter,
- FM.tol = 0.1; % termination criterion,
- FM.ante = 2; % 2 – projected MFS,
- FM.cons = 2; % 2 – weighted LS,

where: FM is the defined structure by Babuska in MATLAB for the parameters of the toolbox.

Table 3 presents the obtained results with the fuzzy nonlinear regression model and the obtained results with the multiple regression model [24].

Table 3

#### Models measurements results with the fuzzy nonlinear regression

Item	Unit	Regression Linear (%VAF)	Fuzzy Clustering (%VAF)
1	05	76.16	100
2	09	97.52	100
3	10	80.01	95.53
4	12	86.89	100
5	13	87.03	99.17
6	15	78.69	100
7	17	87.19	100
8	19	92.66	100
9	22	86.81	100
10	24	94.7	100
11	34	84.93	100
12	35	70.86	99.07
13	38	77.83	100
14	40	94.15	92.28
15	41	92.5	92.6

Figure 9 shows the measured data of the acceleration and the model output data obtained by fuzzy clustering for the vehicle 05.

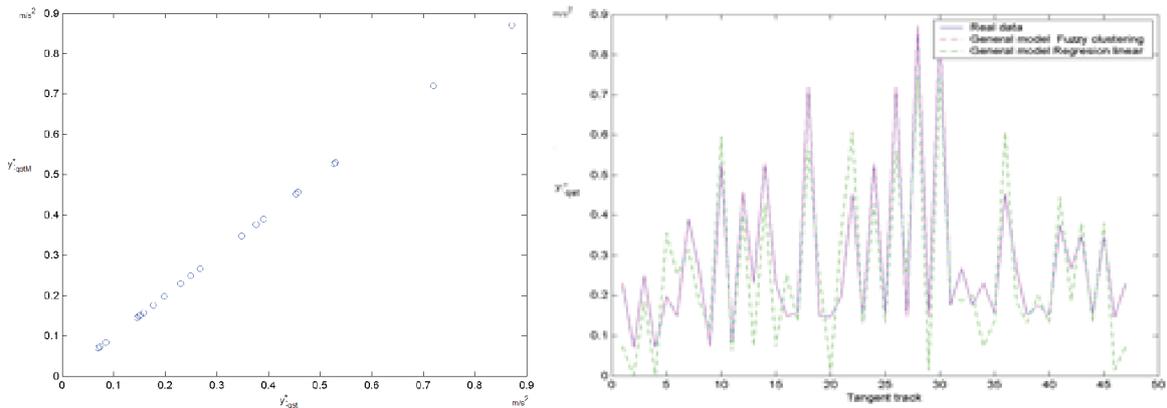


Fig. 9. Plot  $Y_{qst}$  vs. output fuzzy model vehicle 05 and real acceleration curve

It is noticeable that in the table of results there is a VAF of 100%, which corresponds to the line at  $45^\circ$  of the Figure and besides faithfully produced acceleration, as shown in the Fig. 9.

In Table 3, it is observed that the worst VAF coefficient corresponds to the vehicle 40, with a VAF of 92.28. If we plot the data (Fig. 10), it can be clearly observed which model did not estimate the data well, leaving it out of the comparative graphic of  $45^\circ$  (Fig. 10).

This could be due to tuning problems of the model, either in the sensors installation, different geometric conditions of the rail or the equipment capacity, etc. To obtain a general model, all the samples of the 15 vehicles in the matrix were taken, and then a pre-processing consisting of interchanging the files randomly was performed. Afterwards, the same process on each vehicle individually was performed and the results were a VAF of 97.35. It can be graphically observed in Figures 11.

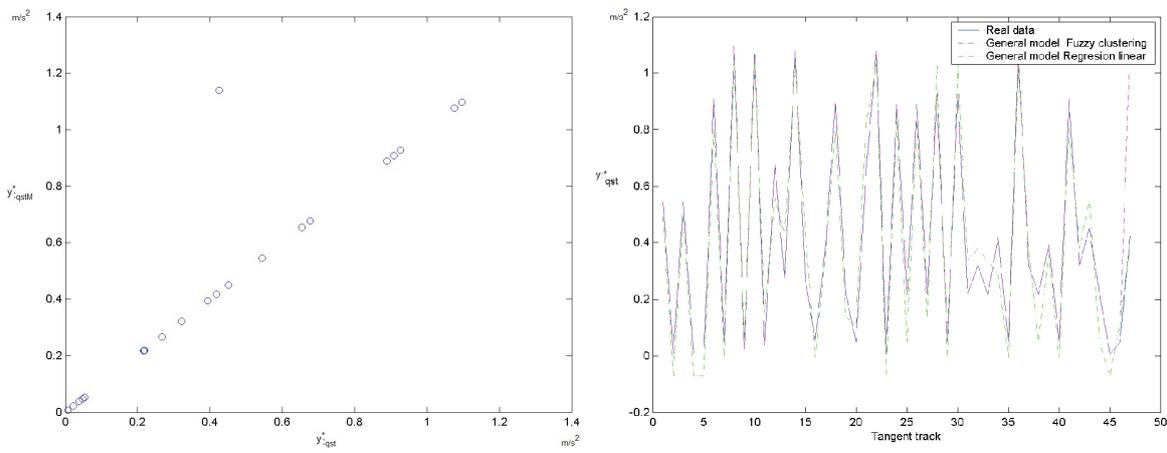


Fig. 10. Plot  $Y_{qst}$  vs. fuzzy model output Vehicle 40 and real acceleration curve

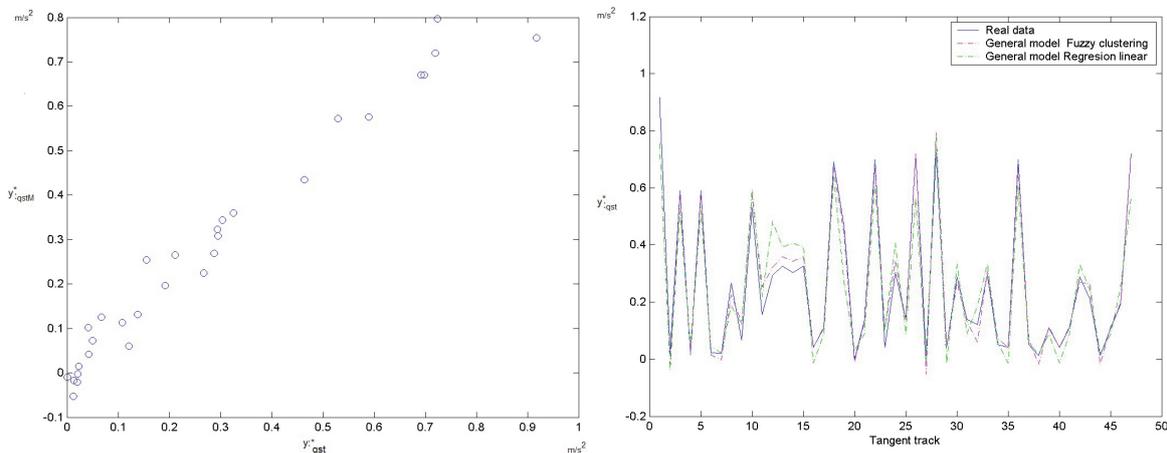


Fig. 11. Plot  $Y_{qst}$  vs. Fuzzy general model output and real acceleration general model curve

## 6. Conclusions

In this article, the main Fuzzy clustering aspects for the model identification were revised.

Although the obtained results with the linear multiple regression are satisfactory, comparing the obtained results, we find that the quality of the fuzzy model is better in 14 out of 15 analyzed vehicles, and only one vehicle of the model of linear multiple regression is better with the fuzzy model.

We showed that Fuzzy clustering is a good tool to approximate nonlinear functions, especially the Takagi-Sugeno model.

This regression model can be integrated into the process for decision support in the maintenance of rail-vehicle interface to reduce the cost associated with the maintenance work, human resources and the increase of system reliability.

Due to the reasons explained before, when it comes to identifying a nonlinear model, we recommend the fuzzy model to be used in future implementations among the SPD.

## Literature

1. Babuska R.: *Fuzzy modeling and identification toolbox*, Delft Centre for System and Control. TU Delft – manual 1996.
2. Babuska R.: *Fuzzy modeling for control*. Kluwer Academic Publishers. Boston 1998, ISBN: 978-0-7923-8154-9.
3. Castañeda L., Żółtowski B.: *Diagnostic system for the metro train*. International Conference on Maintenance Engineering, (ICME), Cheng Du, China, 2006.
4. Castañeda L., Żółtowski B.: *Metoda środowiskowych badań pojazdu szynowego*. Inżyniera i Aparatura Chemiczna, SIMPress, Gliwice, Poland, 2006, pp. 100–104, ISSN 0368-0827.
5. Castañeda L., Żółtowski B.: *Portable Diagnostic System for the Metro Train*. Diagnostyka, vol. 37, Nr 1, (37) 2006, Poland, pp. 39–45, ISSN 641-6414.
6. Chen S., Billings S.A.: *Representation of non-linear system the NARMAX model*. International Journal of Control, 49, pp. 1012–1032.
7. Debiolles A., Oukhelou L., Aknin P.: *Combined use of partial least squares regression and neural network for diagnosis tasks*. Pattern Recognition, 2004, ICPR, 2004, Proceedings International Conference, vol.4, 2004, pp. 573–576.
8. Debiolles A., Oukhelou L., Aknin P.: *Linear and nonlinear regressions using PLS feature selection and NN on defect diagnosis application*. Int. Conf. on Machine Intelligence, ICMI, 2005.
9. Honda K., Ichihashi H.: *Regularized linear fuzzy clustering and probabilistic PCA mixture models*. IEEE transactions on fuzzy system, vol. 13, No 4, 2005 pp. 508–516 ISSN: 1063-6706.
10. Hornik K., Stinchcombe M., White H.: *Multi-layer feed forward networks are universal of approximate*. Neural Networks, vol. 2, Issue 5, 1985, pp. 359–366, ISSN: 1045-9227.
11. Hwang C., Chung-Hoon Rhee F.: *Uncertain fuzzy clustering: inter-vak type-2 fuzzy approach to C-means*. IEEE – transactions on fuzzy system, vol.15, No1, 2007 pp. 107–120. ISSN: 1063-6706.
12. International Union of Railways. UIC, Code 518. Testing and approval of railway vehicles from the point of view of their dynamic behaviour – safety – track fatigue – ride quality. 2nd edition, April 2003, 72 p.
13. Ishida M., Akama M., Kashiwaya K., Kapoor A.: *The current status of theory and practice on rail integrity in Japanese railways – rolling contact fatigue and corrugations*. Fatigue & Fracture of Engineering Materials and Structures, Vol.26, No.10, 2003, pp. 909–919.
14. Kreyszig E.: *Introducción a la estadística matemática: principios y métodos*. México: Editorial Limusa S.A., 1988, 505 p.
15. Kumar M., Stoll R., Stoll N.: *A min-max approach to fuzzy clustering, estimation, and identification*. IEEE transactions on fuzzy system, vol. 14, No 2, 2006 pp. 248–262 ISSN: 1063-6706.
16. Mhaskar H.N.: *Neural networks for optimal approximation of smooth and analytic functions*. Neural computation, vol. 8, 1996, pp. 164–177, ISSN: 0899-7667.
17. Nelles O.: *Nonlinear system identification: from classical approaches to neural networks and fuzzy models*. Springer, Berlin, 2001, ISBN: 3-540-67369-5.
18. Ottomanelli M., Dell'orco M., Sassanelli D.: *A Decision Support System Based on Neuro-Fuzzy System for Railroad Maintenance Planning*. <http://dblp.uni-trier.de>.
19. Specht D.F.: *A general regression neural network*. IEEE transactions on neural networks, vol.2, Issue 6, 1991, ISSN: 1045-9227.
20. Takagi T., Sugeno M.: *Fuzzy identification of systems and its applications to modeling and control*. IEEE Transactions on systems, man, and Cybernetics. Vol. 15, 1985, pp. 116–132.
21. Veaux (de) R.D, Schumi J., Schweinsberg J., Ungar L.H.: *Prediction intervals for Neural Networks via nonlinear regression*. Technometrics, vol. 40, Issue 4, 1998, pp. 273–282, ISSN: 0040-1706.
22. Walpole R.E., MYERS R.H.: *Probabilidad y estadística*. Ed.4, México: McGraw Hill, 1992, 797p, ISBN 968-422-992-5.
23. Zoeteman A., Esveld C.: *State of the art in railway maintenance management: planning systems and their application in Europe*. Systems, Man and Cy-

bernetics, 2004 IEEE International Conference, Vol.5, 2004, pp. 4165–4170.

24. Żółtowski B., Castañeda L., Betancourt G.: *Monitoreo multidimensional de la interfase vía – vehí-*

*culo de un sistema ferroviario*. Congreso Internacional de Mantenimiento – ACIEM – Marzo 2007. Bogotá Colombia.

## Nieliniowy model regresji w kolejnictwie

### Streszczenie

Przenośny system diagnostyczny – SPD ocenia aspekty bezpieczeństwa i jakości biegu pojazdów kolejowych oraz stanu technicznego pojazdu kolejowego. Celem niniejszego artykułu jest oszacowanie nieliniowego modelu regresji związanego z zachowaniem jakości jazdy, przez zastosowanie rozmytego algorytmu klastrowania danych geometrycznych stanu technicznego pojazdu kolejowego i pomiary quasi-statyczne przyspieszeń poprzecznych pojazdów szynowych. Będzie to ocena porównawcza zmierzonego realnego przyspieszenia z przyspieszeniem obliczonym skonfigurowanego modelu dla 15 różnych pojazdów. Uzyskane wyniki będą porównane z wynikami modelu liniowej regresji wielokryterialnej, które były dotychczas w tym celu stosowane.

**Słowa kluczowe:** model regresji, kolejnictwo, zbiór rozmyty

## Нелинейная модель регрессии в железнодорожном транспорте

### Резюме

Портативная система диагностики – ПСД – оценивает аспекты безопасности и качества движения железнодорожных подвижных единиц и технического состояния единицы подвижного состава. Целью этой статьи является оценка нелинейной модели регрессии связанной с сохранением качества движения через употребление нечеткого алгоритма кластеризации геометрических данных, технического состояния единицы подвижного состава и квази-статистических измерений поперечного ускорения единиц подвижного состава. Это будет сравнительная оценка измеренного реального ускорения с ускорением рассчитанным модели сконфигурированной для 15 разных единиц подвижного состава. Полученные результаты будут потом сравнены с результатами модели линейной регрессии нескольких критериев, которые до сих пор использовались для этой цели.

**Ключевые слова:** модель регрессии, железнодорожный транспорт, нечеткое множество