PROBLEMY KOLEJNICTWA
RAILWAY REPORT Issue 185 (December 2019)
DOI: 10.36137/1857E
ISSN 0552-2145 (print) ISSN 2544-9451 (on-line)

Numerical and Experimental Analysis of the Heat Transfer Process in a Railway Disc Brake

Andrzej WOLFF[1], Jacek KUKULSKI[2]

Summary

Whether railway brakes are effective or not substantially depends on the thermal condition of the disc brake and friction linings. An effective research method of the heat transfer in brakes is computer simulation as well as experimental testing on a full-size dynamometric test bench. A 2-dimensional, axisymmetric numerical model of transient heat conduction in the railway brake is presented. Relevant boundary conditions concerning heat generated in the brake and dissipated to the environment are used. The problem is solved with the use of the finite element method. The experimental test and simulation results related to intensive braking of the train from 320 km/h to a halt are preliminary compared. The approximate values of maximum temperatures at the end of braking are obtained – ca. 500°C with a certain divergence of temperature profiles during the process under consideration. It is advisable to further develop research concerning this problem.

Keywords: dynamometric test bench, heat transfer in brakes, simulation investigations

1. Introduction

1.1. Thermal phenomena in brakes

The operation of railway and car brakes is inseparably accompanied by thermal processes. Practice shows that they have a significant impact on the operation of brakes and intensity of friction pair wear, and also contribute to the damage of cooperating elements. A considerable influence on the course of tribological phenomena emerging at the contact of friction pairs is attributed to temperature. With its substantial growth, the friction coefficient changes and usually its value drops. In effect, the braking effectiveness declines [1, 7, 13, 15, 17]. In addition, the resistance of cooperating elements to abrasive wear lowers. Subject to considerable thermal loads, the result may also be the structural and chemical degradation of the frictional material [1, 13, 15]. In turn, thermal expansion is accompanied by deformation and heat stresses leading to short-term disruption of the cooperation between friction elements. As a result of cyclical stress, the material of a disc brake may break, starting from friction surfaces and going deeper into the material [1, 8].

Bearing the above in mind, the thermal phenomena which accompany the operation of brakes are subject to numerous studies, both theoretical and experimental [1–13,15–19].

1.2. Railway brakes testing

The braking system is one of the major solutions influencing the operational reliability and safety of rail vehicles. It is used in all types of vehicles. The operation of braking systems in challenging conditions, related to the growth of operating speed, loads and frequency of operation, leads to a need to include effects related to structure dynamics in the design process. The rise in driving speed and vehicle weight as well as the use of new materials for elements of friction pairs result in new obstacles in braking systems.

As a result of friction occurring in disc brakes, mechanical energy converts into thermal energy. With regard to braking systems used in mechanical vehicles (road and rail vehicles), this phenomenon results from frictional interaction between the brake lining and surface of the disc brake. The heat accumulated in frictional elements of such units leads to thermal disturbances which, in turn, result in the emergence

[1] Ph.D., Eng.; Warsaw University of Technology, Faculty of Transport; e-mail: wolff@wt.pw.edu.pl.
[2] Ph.D., Eng.; Railway Research Institute, Rolling Stock Testing Laboratory; e-mail: jkukulski@ikolej.pl.

of unfavorable phenomena, such as hot spots, cracks, etc. [5, 6].

Many researchers have dealt with the analysis of braking systems. The authors of one work [5] presented their research results obtained from a real rail vehicle braking system, showing the generation of hot areas during tribological tests of friction pairs of railway brakes. Such phenomena reduce the effectiveness of braking and cause chemical changes in the structure of braking elements made of organic materials. Another work [6] was dedicated to the study of heat distribution caused by friction in braking systems in high-speed rail vehicles. In [4], in turn, the author presented bench tests of sintered brake linings from the initial speed of 300 km/h; tribological results of the frictional material covered by these studies were demonstrated there.

2. Heat transfer model in railway brakes

2.1. Mathematical model of heat conduction

The heat transfer in the brake should be considered as transient process. This is the case due to the fact that the following are variable in time: heat generated during braking and dissipated to the environment, as well as the temperature field of brake frictional elements. he primary task is to choose a suitable mathematical model of heat transfer in railway brakes (Fig. 1). In practice, a major role is played by the heat conduction through the brake rotor (here: disc).

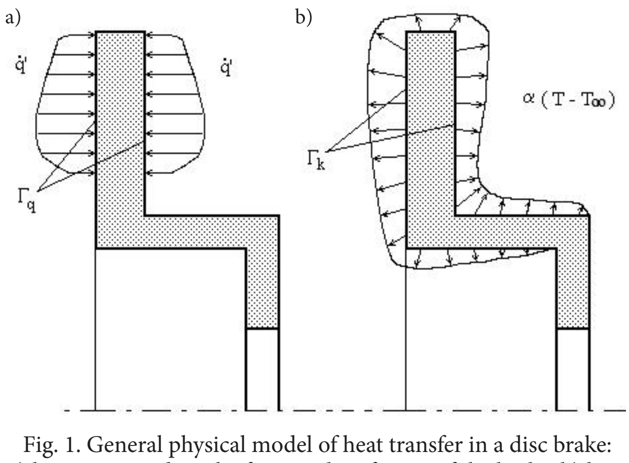

Fig. 1. General physical model of heat transfer in a disc brake: a) heat generated on the frictional surface Γ_q of the brake, b) heat dissipated to the environment from whirling surfaces Γ_k [18]

Analyzing the geometric shapes of full (i.e. non-ventilated) railway brake discs, we can assume that they are axisymmetric. Such symmetry does not apply to boundary conditions of the heat transfer problem under consideration. It is necessary to note that friction pads, at the contact with which the heat flux is generated, cover only a part of the disc perimeter (brake rotor). Apart from this, the pressure distribution on the perimeter of the disc is uneven, and therefore the heat flux is not uniformly distributed. We need to remember, however, that the rotor moves in relation to linings. For this reason, the assumption of axisymmetry of the heat flux on frictional surfaces means only time – averaged boundary conditions in the period corresponding to one rotation of the disc brake. This kind of simplification ends up with a minor mistake, particularly at high rotational speeds of the brake rotor.

For the aforesaid reasons, a two-dimensional, axisymmetric model was chosen for simulation studies of the heat transfer phenomena in railway brakes.

In this case, the heat conduction equation will take the following form [14]:

$$\rho c \frac{\partial T}{\partial t} = \frac{1}{r}\left[\frac{\partial}{\partial r}\left(r\lambda\frac{\partial T}{\partial r}\right) + \frac{\partial}{\partial z}\left(r\lambda\frac{\partial T}{\partial z}\right)\right], \quad (1)$$

where:
- ρ, c, λ – mass density, specific heat and thermal conductivity of the rotor,
- r, z – cylindrical coordinates,
- T – temperature,
- t – time.

The heat flux q is generated on friction surfaces of the rotor Γ_q (Fig. la). Here are boundary conditions of the II type, in the form [14–18]:

$$-\lambda\frac{\partial T}{\partial n} = \dot{q}'(r,z,t), \quad (2)$$

where:

$$\dot{q}' = \xi\,\dot{q} \text{ and } \dot{q} = \mu\, p\, v, \quad (3)$$

where:
- ξ – coefficient of heat flux distribution between the rotor and friction linings,
- μ – friction coefficient,
- p – pressure (between the rotor and brake lining),
- v – slip velocity.

The heat transfer with the environment occurs on the free surfaces Γ_k (Fig. lb). This is described by boundary conditions of the III type, in the form [14–18]:

$$-\lambda \frac{\partial T}{\partial n} = \alpha \left[T(r,z,t) - T_{\infty} \right] \tag{4}$$

where:

$$\alpha = \alpha_k + \alpha_{pr}, \tag{5}$$

where:

α_k - convection-based heat transfer coefficient,
α_{pr} - radiation-based heat transfer coefficient,
T_{∞} - ambient temperature.

The form of the initial condition for the problem under consideration is as follows [14–18]:

$$T(r,z,t_0) = T_0, \tag{6}$$

where: T_0 – initial temperature.

2.2. Boundary conditions of the problem

Integral parts of every mathematical model are boundary conditions. First of all – heat flux distributed between a disc (brake rotor) and friction linings is generated on frictional surfaces. The biggest part of this flux is transferred to the rotor due to the considerable difference between:

1) values of thermal parameters of friction pair materials,
2) areas of active frictional surface of both elements of the pair [15–18].

The heat flux generated in the brake examined on the inertia test bench was established in the way specified in subsection 4.2 of this article – equation (7).

Secondly – there is a complex heat transfer with the environment on the free surfaces of the brake, based primarily on convection, and to a lesser extent on radiation. The impact of radiation rises substantially only at very high temperatures (range: 300–500°C). A highly difficult task is to establish the value of the coefficient of the convection-based heat transfer with the environment. This is mainly the result of the complexity of conditions of the brake flow by the cooling air, as well as the variability of these conditions. As a rule, the above-stated parameter is determined with the use of criterion formulas of similarity theory [14–18]. Our own computer software for generating boundary conditions of the II and III type, called *GENTGV,* has been developed, eq. (2)–(5).

Interestingly, in the last couple of years, the flow around brakes and convection-based heat transfer have been modelled with the use of CFD software (*Computational Fluid Dynamics)* [10, 12].

2.3. The numerical method used to solve the problem

In the first place, the possibility of solving the issue of thermal conductivity in brakes with the use of three numerical methods was studied: the finite difference, finite element and boundary element methods [14, 20]. As a result of the aforesaid analysis, the finite element method was chosen. This was primarily due to the following factors: the all-purpose nature of its algorithm and widespread presence, as well as the capability of precise approximation of the edge of the analyzed objects. Elements of axisymmetric shape were used. In their axial sections, these are four-sided, 8-node elements, with straight or curved edges (2-order isoparametric elements) [15, 20]. For the purposes of analysis of the heat transfer in brakes, the relevant computer program of the finite element method called FEMHEAT was developed [15].

3. Experimental testing

3.1. Dynamometer stand

The experimental tests were conducted on the special inertial braking bench intended to test friction pairs of brakes of rail vehicles of the Railway Institute. The test bench has been certified by UIC (International Union of Railways) for all-purpose braking benches with the max. speed up to 420 km/h. The design of the test bench allows friction pairs of railway brakes, dedicated to high-speed fixed-formation trains, train units, locomotives and rail buses of natural size corresponding to reality, to be tested. Fig. 2 shows the stationary test stand as well as a test cabin view.

The basic technical parameters of the inertial test stand are presented in Table 1.

3.2. Research program

The research procedure encompassed bench tests in a steel axial disc without ventilation of the following dimensions: 640x45 mm. These discs were used in the first TGV trains manufactured by *Alstom*. The tests adopted sintered brake linings with a total contact surface with the disc brake of 400 cm^2 (Fig. 3a). Table 2 depicts selected research programs for the full disc brake mounted on the axis (without ventilating ducts).

Braking no. 2 was completed shortly after braking no. 1 without any additional procedure for cooling the disc brake down. Cooling took place when gathering speed with a constant acceleration of 0.3 m/s^2 to reach the required speed of 320 km/h. Due to the lack of ventilating ducts, the temperature of the full friction disc surface was measured with the use of special slip thermocouples (Fig. 3b).

a) b)

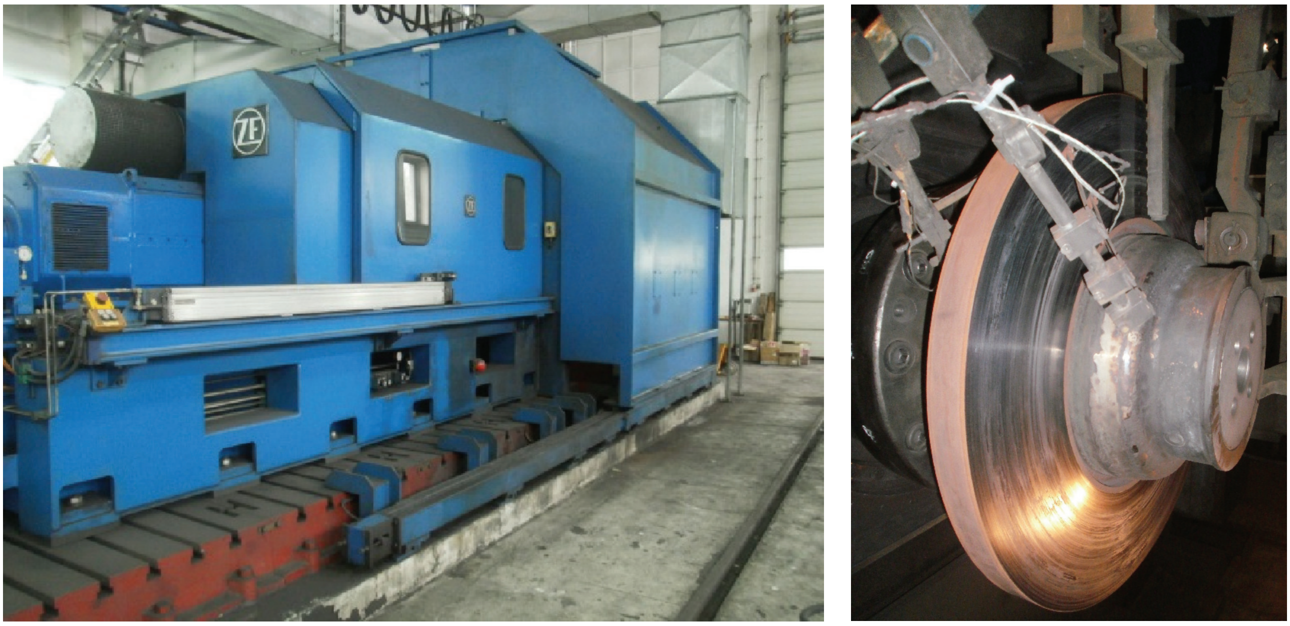

Fig. 2. Stationary test stand: (a) general view, (b) test cabin [photos by J. Kukulski]

Table 1

Basic technical parameters of the inertial test stan

Item	Parameter	Value
1.	Range of vehicle speed (for wheel Ø 890 mm) [km/h]	3.5÷420
2.	Maximum rotational speed [r./min.]	2500
3.	Driving motor power at 1150 r./min [kW]	536
4.	Torque up to 1150 r./min. [Nm]	4450
5.	Max. braking torque: – braking to a halt [Nm], – continuous braking [Nm].	 3000 4450
6.	Range of moments of mass inertia with electric simulation [kgm^2]	150÷3000
7.	Maximum simulated mass per friction pair [t]	15
8.	Range of total pressure force adjustment for brake shoes in the disc brake [kN]	0÷60
9.	Range of temperatures for the disc brake (road wheel) [°C]	0÷1000

[Own study].

Table 2

Research program for full disc brake mounted on the axis

	Test parameters			
Test no.	V_{max} [km/h]	pressure force of brake linings F [kN]	braking weight m [kg]	temperature at the outset of braking T_a [°C]
Braking 1	320	14/18	4000	60
Braking 2	320	19/25	4500	150

[Own study].

3.3. Test results

The results of experimental tests of two cases of braking are demonstrated in Table 3 and in Figs. 4–7.

The figures illustrate registered parameters, such as average temperatures on the surface of the disc from six slip thermocouples, the pressure of the linings on the friction surface of the disc, braking energy, as well

a) b)

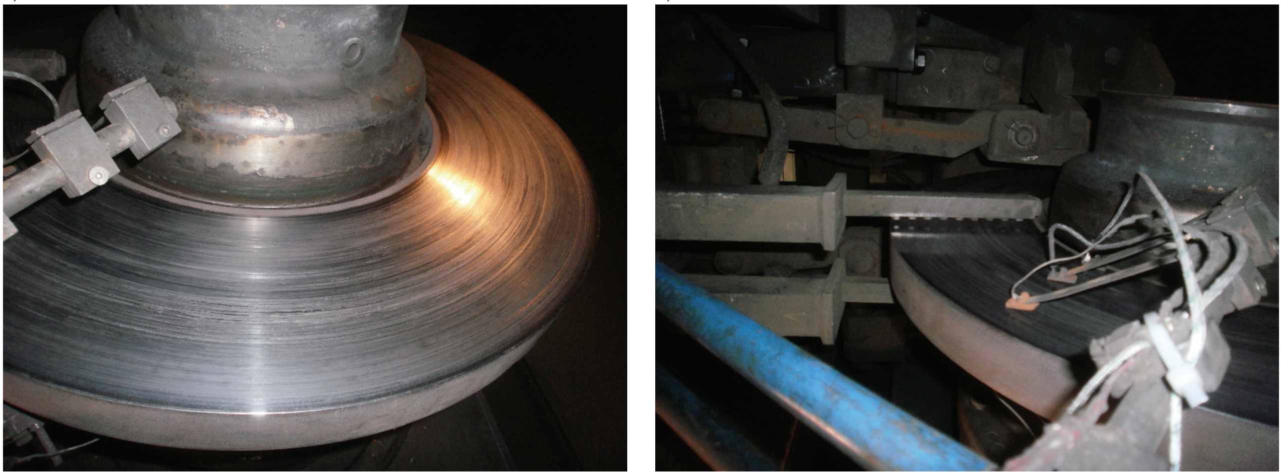

Fig. 3. Research object: a) brake disc and linings, b) sliding thermocouples [photos by J. Kukulski]

as temporary friction coefficient in the linear speed of the whirling object.

Analyzing the diagrams above, you can notice that throughout braking the temperature rises almost linearly to reach the maximum value. A distinctive effect was not recorded; the effect was concerned with a slight drop in temperature of the disc friction surface at the end of the braking process (with low speed and amount of heat generated). This may arise from imperfect slip thermocouple measurements.

Table 3

Results of experimental tests of two cases of braking

Test no.	V_p [km/h]	Braking energy [MJ]	Braking power [kW]	Braking distance [m]	Average friction coefficient	Mean value T_{max} [°C]
1	320	15.45	186	6859	0.278	338
2	320	17.19	257	5287	0.297	465

[Own study].

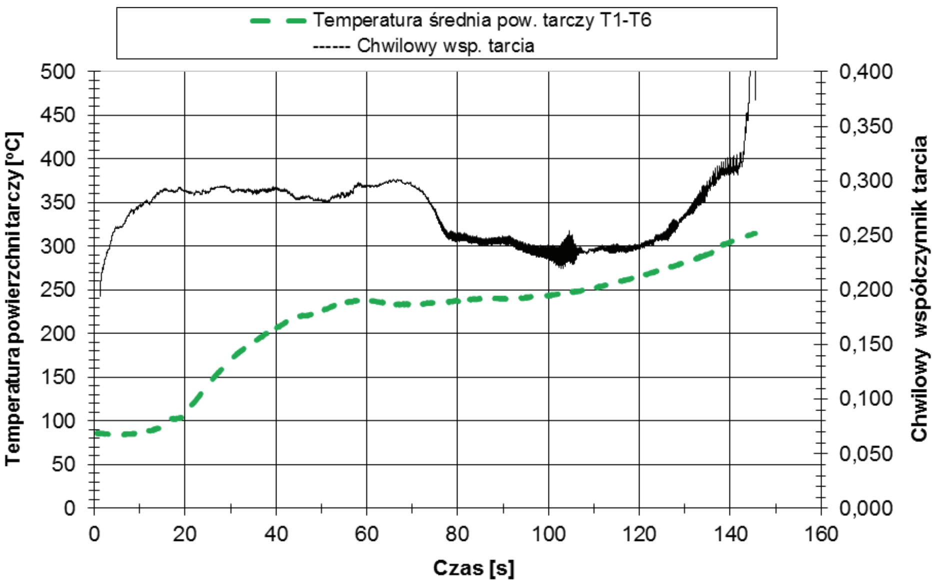

Fig. 4. Friction coefficient and average temperature of the disc surface versus time for braking from V = 320 km/h (braking no. 1) [own study]

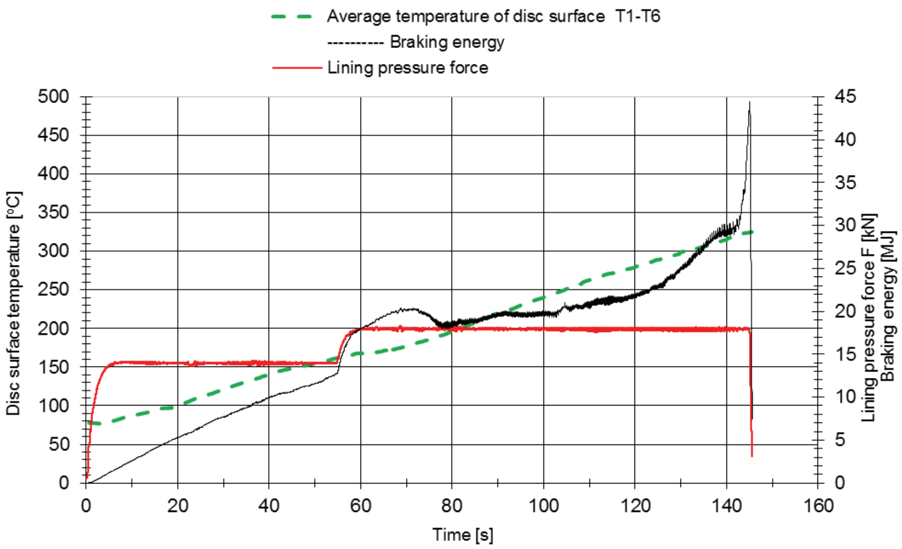

Fig. 5. Pressure force of linings, average temperature of disc surface and braking energy versus time for test braking from $V = 320$ km/h (braking no. 1) [own study]

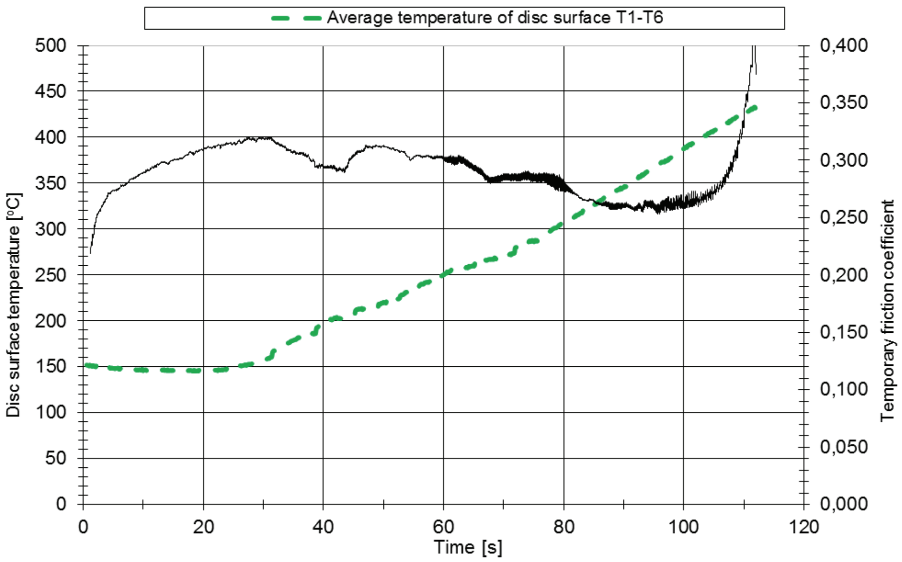

Fig. 6. Friction coefficient and average temperature of disc surface versus time for braking from $V = 320$ km/h (braking no. 2) [own study]

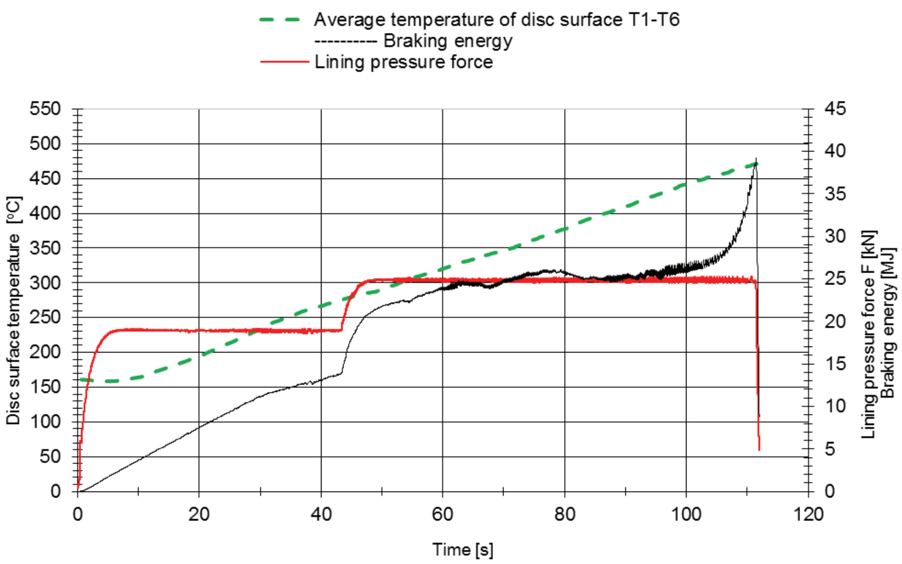

Fig. 7. Pressure force of linings, average temperature of disc surface and braking energy versus time for test braking from $V = 320$ km/h (braking no. 2) [own study]

4. Simulation investigations

4.1. Research object

The simulations were performed for the disc brake of a high-speed rail vehicle (Fig. 8). The basic parameters of this brake are presented in Table 4.

It is important that we emphasize the fact that one wheel of the rail car axle applies to two brake discs. For this reason, a symmetry plane can be found on the right side of Fig. 8a.

Fig. 8b shows the mesh of finite elements of the disc (without disc hub) initially adopted. Further analyses are intended to supplement the mesh to include skipped parts connected to the disc brake. Fig. 8b distinguishes also mesh nodes in which the temperature variations over time were calculated. These points apply to frictional surface and 4 different depths (4, 8 and 15.25 mm) under this surface up to the center of the disc thickness (22.5 mm).

In turn, Fig. 8c presents the location of slip thermocouples of the brake under consideration (see Fig. 3). In further analyses, it is planned to slightly correct the mesh of finite elements so that the points of thermocouple location coincide with the location of corner nodes, or nodes in the middle of the edges of elements.

4.2. Results of simulation investigations

The purpose of numerical simulations was to reconstruct the heat transfer conditions available during previously described (point 3) experimental tests of the disc brake on the inertial test bench.

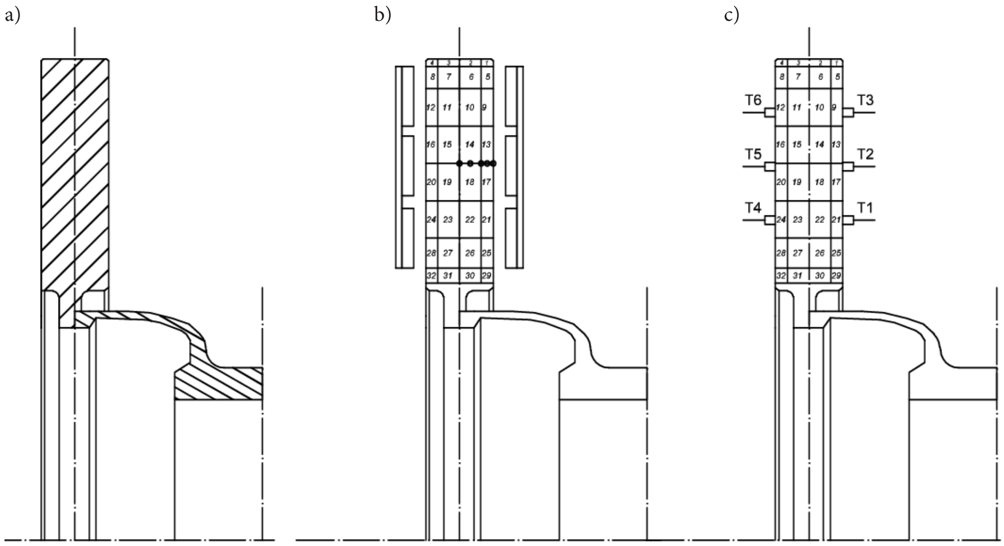

Fig. 8. Axial section of a high-speed train brake disc: a) overview drawing, b) marking of friction lining and initially adopted mesh of finite elements of the brake rotor (without disc hub), c) location of thermocouples of the disc under consideration [own study]

Table 4

Technical data of rail vehicle brake

General parameters of the brake		
External diameter of the disc brake [mm]	640	
Thickness of the disc brake [mm]	45	
Frictional surface	**Brake rotor**	**Friction linings**
Disc brake [m^2]	0.2099	0.0400
Material parameters	**Steel disc brake**	**Friction linings**
Heat conduction coefficient [W/mK]	37.2	2.4
Density [kg/m^3]	7300	5250
Specific heat [J/kgK]	615	1600

[Own study].

The calculations were made for braking no. 2 (Tab. 2). Figure 9 shows the time trends for train forward speed and angular velocity of the disc brake reconstructed on the inertial test bench. In the case of intensive braking until a halt, both values decline almost linearly over braking time. The initial speed of the rail vehicle is high and equals 320 km/h, which applies to an angular velocity of the disc brake ca. 200 rad/s.

The inertial test bench ensured sufficiently high brake thermal loads, similar to the ones existent when the rail vehicle brakes in real conditions and similar sliding speed of brake friction pairs [5, 6].

The heat transfer rate generated in the brake $Q'(t)$ was established on the basis of known time trends of braking torque $M_H(t)$ and angular velocity $\omega(t)$ of the inertial bench shaft, and therefore the disc brake under consideration:

$$\dot{Q}(t) = M_H(t)\cdot\omega(t) - F_r(t)\cdot r_r\cdot\frac{\pi\cdot n(t)}{30} \tag{7}$$

where:

$F_r(t)$, $n(t)$ – measured time trends of the force on the reaction arm r_r and rotational speed of the test bench shaft.

Next, the time trend for heat flux q'(t) which penetrates the disc was established, equation (3). The size of the friction surface of the rotor and linings as well as the distribution of the heat flux at the contact of friction pairs were taken into account. The time trends of $Q'(t)$ and $q'(t)$ are depicted in Fig. 10.

Due to the two-stage braking of the rail vehicle, near 44 s a clear growth in the heat flux generated in the brake can be observed. Then, both $Q'(t)$ and $q'(t)$ drop almost linearly to zero as the braking time passes. Fig. 11 shows the time trends for the heat transfer coefficients α_i on free surfaces (Γ_{ki} , $I = 1,\ldots,4$) of the disc brake.

On the right side of the diagram we can find the assumed free surfaces Γ_k of the disc brake, from which heat flows out to the environment. The dependencies concerning the coefficients of heat transfer on these surfaces (disc, external and internal cylindrical) were given in another work [15]. The heat transfer coefficient values (Fig. 11) decline along with a drop in rail vehicle speed, and consequently a drop in the brake flow-round intensity. Interestingly, in the preliminary analysis of the heat conduction through the brake rotor, the disc hub was not taken into account. The heat transfer to this area was replaced with a suitable outflow into the environment through the internal cylindrical surface $\Gamma_{k4.}$

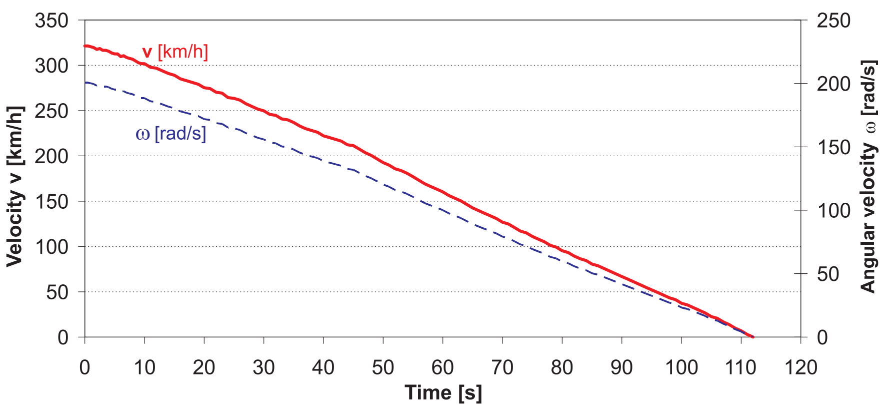

Fig. 9. Simulated train forward speed and angular velocity of the disc brake tested on the inertial bench versus time. Braking from the initial speed of v_0 = 320 km/h until a halt (braking no. 2)

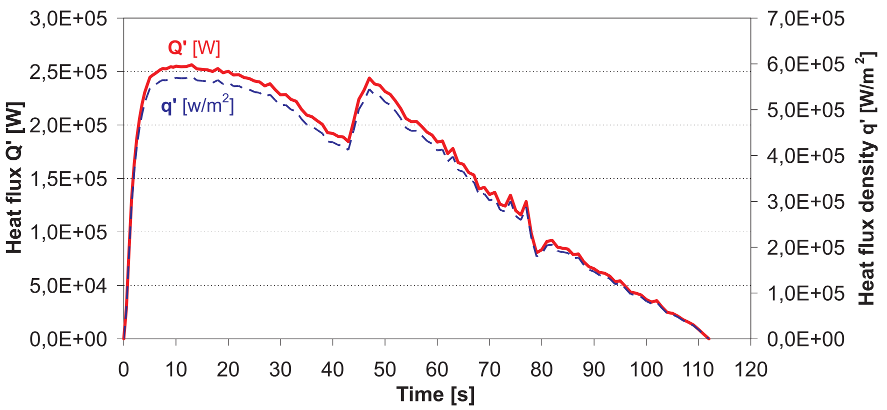

Fig. 10. Heat transfer rate Q'[W] generated in the disc brake and heat flux q' [W/m2] on its frictional surface versus time. Two-stage braking from the initial speed of v_0 = 320 km/h until a halt [own study]

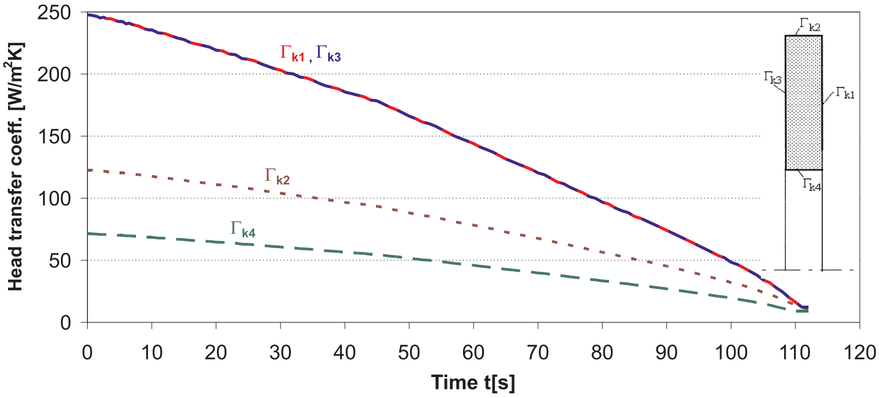

Fig. 11. Heat transfer coefficients αi on free surfaces (Γ_{ki} , I = 1,…,4) of the disc brake vesus time. Braking from the initial speed of v_0 = 320 km/h until a halt [own study]

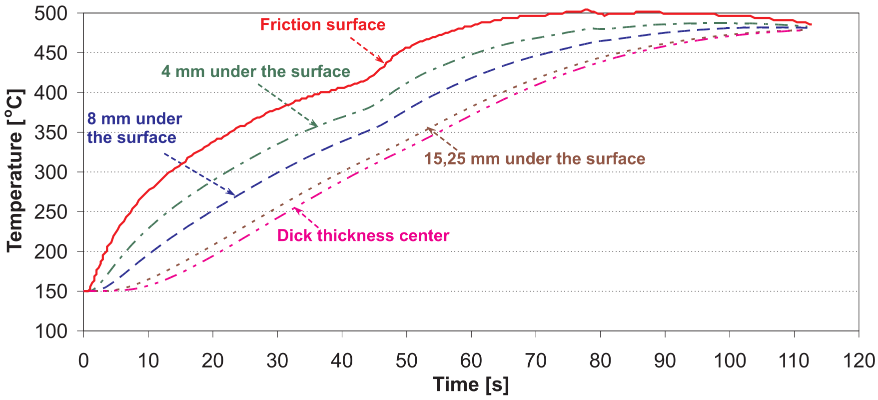

Fig. 12. Temperatures Ti at selected points (Fig. 8b) of the disc brake versus time. (The calculated temperatures concern frictional surface and 4 various depths under this surface, up to the center of disc thickness). Vehicle braking with the average deceleration a_h ≈ 0.8 m/s2 from the initial speed of v_0 = 320 km/h until a halt. The disc initially heated up to the temperature of T_0 ≈ 150°C [own study]

The time trends presented in Fig. 11 served as boundary conditions of the III type in the developed heat conduction model.

Fig. 12 shows the time trends for temperatures at 5 chosen points of the disc section (Fig. 8b), including the friction surface. As expected, the temperature trends at all points are nearly monotonous. The fastest growth in temperature is observed on the disc friction surface where heat is generated. The further from this surface, the milder the growth of temperature at this point. The calculated temperatures apply to frictional surface and 4 different depths under this surface (Fig. 8b) up to the center of disc thickness. In the final braking stage, when a small amount of heat is generated, the temperature on the frictional surface drops slightly. The outflow of heat to further parts of the brake rotor rises.

Such a nature of temperature trends in disc brakes has been confirmed in many works, including some by one of the authors of this publication [15–18].

5. Comparison of simulation and experimental test results

Fig. 13 presents the time trends for temperatures on two friction surfaces of the disc, measured by 6 slip thermocouples (Fig. 8c). The calculated time trend for the temperature in the center of the frictional surface, that is, near the thermocouples T_2 and T_5 (Fig. 8c), is illustrated. Simultaneously, the results of experimental tests and simulations were subject to preliminary comparison. Similar values of maximum temperatures at the end of braking were achieved – ca. 500°C. We can clearly observe a certain divergence of time trends for temperatures during the process under consideration. This may result from imperfect slip thermocouple measurements. It is advisable to develop further studies of this issue.

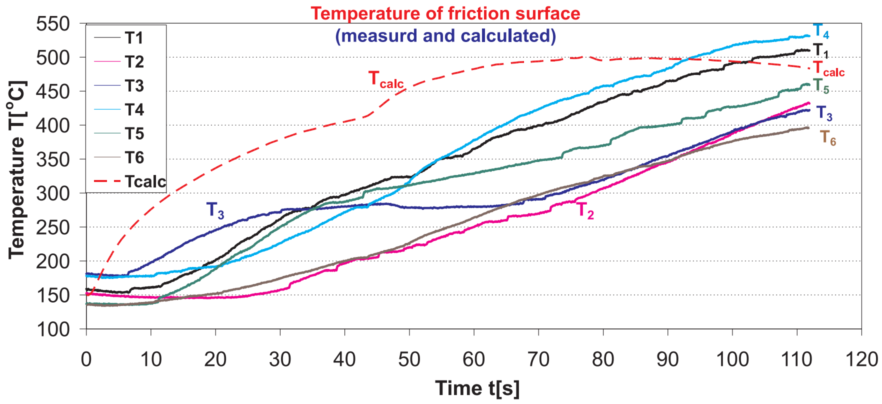

Fig. 13. Comparison of temperatures (measured and calculated) of friction surface of the disc brake during braking. Measured temperatures (T_i, I = 1,…,6) apply to the location of thermocouples (Fig. 8c), and the calculated temperature Tcalc applies to the mesh node on the friction surface (Fig. 8b). Braking from the initial speed of v_0 = 320 km/h until a halt. The disc initially heated up to the temperature of $T_0 \approx 150$°C [own study]

6. Conclusions

The experimental tests and simulations carried out serve as an introduction to further research and are aimed at using the authors' application in testing the railway brake. The tests will be continued in order to check other variants and assumptions related to friction pairs.

Based on the comparison of the results of bench tests and simulations, similar values of maximum temperatures at the end of braking were obtained, that is ca. 500°C. However, certain divergence in the time trends of temperatures during the process under consideration was recorded. This may result from imperfect slip thermocouple measurements or material assumptions of the simulation model.

The tests on the inertial bench and simulation program can be used to analyze the heat transfer in railway brakes, assess or forecast changes in braking effectiveness due to the thermal condition of brakes and optimization of the brake design.

Literature

1. Eriksson M., Filip Bergman F., Jacobson S.: *On the nature of tribological contact in automotive brakes*, Wear 252, pp. 26–36, 2002.
2. Hwang P., Wu X., Jeon Y.B.: *Thermal-mechanical coupled simulation of a solid brake disc in repeated braking cycles*, Proc. Inst. Mech. Eng., Vol. 223, Part J: J. Engineering Tribology, pp. 1041–1048, 2009.
3. Jung S.P. et.al.: *Finite element analysis of thermoelastic instability of disc brakes*, Proc. of the World Congress on Engineering, 2010, Vol II, London, 2010.
4. Kim M.: *Development of the Braking Performance Evaluation Technology for High-speed Brake Dynamometer*, Intern. Jour. of Systems Applications, Engineering & Development, Issue 1, Vol. 6, 2012.
5. Konowrocki R. et.al.: *Temperature field analysis of brake discs for high speed train using infrared technology*, Advanced Rail Technologies – 5th Intern. Conf., Warsaw, 2016.
6. Konowrocki R. et.al.: *Dystrybucja energii cieplnej w elementach układu hamulcowego pojazdów dużych prędkości*, [*Heat energy distribution in high-speed vehicle braking system elements*], Pojazdy Szynowe, Vol.2, pp.1–14, 2014, Supl. on CD.
7. Lozia Z., Wolff A.: *Thermal state of automotive brakes after braking on the road and on the roll-stand*, SAE Special Publication No 1229 / SAE Technical Paper No 971040, pp. 107–115, Detroit (USA), 1997.
8. Mackin T.J. et al.: *Thermal cracking in disc brakes*, Engineering Failure Analysis 9, pp. 63–76, 2002.
9. Nisonger R.L., Yen Chih-Hung, Antanaitis D.: *High temperature brake cooling – characterization for brake system modeling in race track and high energy driving conditions*, SAE Paper 2011-01-0566, 2011.
10. Palmer E. et.al.: *Analysis of air flow and heat dissipation from a high performance GT car front brake*, SAE Paper No. 2008-01-0820, 2008.
11. Shahzamanian M.M. et.al.: *Transient and thermal contact analysis for the elastic behavior of functionally graded brake disks due to mechanical and thermal loads*, Materials and Design 31, pp. 4655–4665, 2010.
12. Schuetz T.: *Cooling analysis of a passenger car disk brake*, SAE Paper 2009-01-3049, 2009.
13. Tonchev A., Hirschberg W., Jagsch S.: *Investigation of the thermal vehicle brake behavior during the vehicle's development phase by co-simulation*, SAE Paper No. 2007-01-3935, 2007.
14. Wiśniewski S.: *Wymiana ciepła* [Heat exchange], Wydawnictwo Naukowo-Techniczne (wyd. 6), Warszawa, 2009.
15. Wolff A.: *Teoretyczno-eksperymentalna analiza procesu wymiany ciepła w hamulcach samochodowych* [Theoretical and experimental analysis of the heat transfer in car brakes], doctoral dissertation, Politechnika Warszawska, Wydział Transportu, Warszawa, 1992.

16. Wolff A.: *A method to achieve comparable thermal states of car brakes during braking on the road and on a high-speed roll-stand*, Archives of Transport, Vol. XXII, No.2, pp. 259–273, 2010.
17. Wolff A.: *Analiza numeryczna i eksperymentalna procesu wymiany ciepła w hamulcach samochodowych* [Numerical and experimental analysis of the heat transfer in car brakes], Postępy Nauki i Techniki, Lublin, 2012, No. 14, pp. 250–261.
18. Wolff A.: *Modelowanie i symulacja numeryczna procesu wymiany ciepła w hamulcach samochodowych* [Modeling and numerical simulation of the heat transfer in car brakes], Prace Naukowe Politechniki Warszawskiej, Politechnika Warszawska, Wydział Transportu, 2016, No. 112, pp. 411–421.
19. Yildiz Y., Duzgun M.: *Stress analysis of ventilated brake discs using the finite element method*, Intern. Journal of Automotive Technology, 2010, Vol. 11, No. 1, pp. 133−138.
20. Zienkiewicz O.C., Morgan K.: *Finite Elements and Approximation*, John Wiley & Sons, Inc. NY, 1983.