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Speed of Trains Running on Parallel Track Connections Using Curved Turnouts

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Summary
The issue of connecting parallel main tracks located in a circular curve by means of curved turnouts is discussed in the paper, focusing on determining the achievable train speed. Selected geometries are used in the analysis. The curved turnout diverging track radii and the corresponding train speeds are determined. An analytical notation is used, thereby creating greater options in specific applications. It is shown that the speed of trains running on parallel track connections depends on the type of basic turnout subjected to curving (i.e. mainly on the radius of this turnout), while the second very important factor is the value of the track cant used. In each case, the determined speed resulting from the diverging track radius is lower than the speed on the main tracks. While discussing the general principles of constructing parallel track connections in a circular arc using curved turnouts, it is pointed out that in some situations the speed of travel must be further reduced due to the need to connect the ends of the diverging tracks with a circular arc.

Keywords: railway turnouts, turnout curving, speed analysis, connecting diverging tracks

1. Introduction

The problem of turnouts, as described in many publications, such as [1-2, 17, 20, 22, 24], usually pays less attention to the issues related to the turnout curving and the use of curved turnouts connecting the tracks in a circular arc. This topic is presented only in some publications [3, 10, 15, 19, 21]. The railway standards and regulations in force [4, 8, 26] introduce significant application restrictions for curved turnouts. They may be used only in economically justified cases, i.e. when the use of standard turnouts would result, among others, in the introduction of a local speed limit, excessive lengthening or shifting of railroad switches or train service stations, extra earthworks, the need to obtain new land (resulting from the need to shift or change the course of the track system) or a collision with existing infrastructure (e.g. engineering structures).

Despite these application restrictions, the problem of curved turnouts is an important railroad structure issue. It is universal in nature and must be treated in the same way by every railway management organisation. This, of course, also applies to problems relating to the theoretical foundation of the issue described.

In Poland, for several years, the basic and most widely-recognised study dealing with the issues of curved turnouts was the book by Władysław Rzepka entitled “Rozjazdy łukowe w planie i profilu” [Curved turnouts in plan and profile] [23]. It is, in fact, a unique piece of work, which addresses this problem in a comprehensive manner and analyses numerous cases under consideration. Each considered situation has its own geometrical scheme, which is the basis for determining the appropriate symbolic notation. It should be emphasized that the publications that appeared in the following years also contained elements of the discussed issue, but did not bring any new knowledge. The authors of those publications treated the work of W. Rzepka [23] as a model and as the only reference. This applies both to the last edition of one book [16] and, for example, to the publication [27] from 2015.

The breadth of the above-mentioned study, in which many different cases were analysed, prompts a question regarding the extent to which its author used his own thoughts and the solutions of other authors. After so many years, an answer to these questions is not possible here, but, judging from the references, it presents the latest technology of the time, which is largely a presentation of the achievements of the thirties and forties. The author quoted specific German works [5-7, 9, 18, 25] and referred to unspecified articles in the following magazines: “Bahning-

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enieur", "Eisenbahnbau", "Eisenbahn" and "Organ für die Fortschritte des Eisenbahnwesens".

There is one characteristic regularity in the achievements to date. Appropriate theoretical relationships are determined on the basis of geometrical schemes, using their basic properties (circles, right-angled triangles, and triangular similarities). However, they lack elements of analytic geometry, even the trigonometric function is avoided in the obtained formulas. Therefore, an irresistible conclusion is drawn that the developed theory was adapted to the computational possibilities of that time.

A radical change in the approach to the described issue is presented in [12–14]. An analytical notation was used to determine the appropriate geometric parameters of a curved turnout. Three possible options of turnout curving were considered in detail:

1) One-sided curving, in which the main track curve with the radius R is consistent with the diverging track curve with the radius R_0 in the standard turnout (variant I);
2) Two-sided curving, in which the curve of the main track with the radius R is directed opposite to the curve of the diverging track with the radius R_0 in the standard turnout, with the condition $R > R_0$ (variant II);
3) One-sided curving, in which the curve of the main track of the turnout with the radius R is directed opposite to the curve of the diverging track with the radius R_0 in the standard turnout, with the condition $R < R_0$ (variant III).

2. General rules for connecting main lines in a circular arc

Curved turnouts are created by the curving of both tracks of an ordinary turnout with a design that makes this operation possible (called a standard turnout). The main use of curved turnouts is to connect with parallel main lines located in a circular arc. The process of constructing such a track geometry includes the following stages:

- Selection of standard turnouts for curving for both main lines (outer and inner);
- Appropriate curving of selected turnouts;
- Determining the proper location of both curved turnouts in the inner and outer tracks;
- Connecting the diverging tracks of curved turnouts with straight and curved sections.

During curving, the turnout bevel triangle ($A_1B_1C_1$ in Fig. 1 and $A_2B_2C_2$ in Fig 2) retains the same dimensions as in the standard turnout, but changes its position (it is rotated around the apex A_1 or A_2). The ends of the main track and the diverging track must be on the guide wheel of a turnout with a centre that overlaps with the centre of the curved standard turnout (i.e. point A_1 or A_2).

In the outer track (with a slightly higher value of circular arc radius R_{out}) a turnout is used which is curved according to the direction of its diverging track – variant I of curving [13]. The diverging track radius is reduced, and it is this radius that determines, to a large extent, the achievable train speed on the connection of the two main lines. For this reason, it will be most advantageous in this case to use the standard turnout with the largest possible radius in the diverging track for curving.

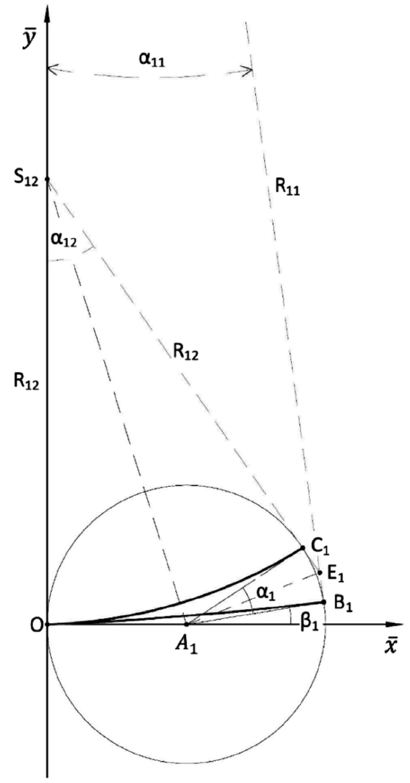

Fig. 1. Standard turnout scheme in variant I of curving: O – turnout toe, A_1 – standard turnout centre, B_1 – main track end, C_1 – diverging track end, S_{12} – diverging track curve centre [author's study]

In the inner track (with a lower value of the circular arc radius R_{inn}) a turnout is used which is curved according to the direction of its diverging track – variant III of curving (Fig. 2) [13]. The diverging track radius is significantly increased. Such a curving method guarantees that the position of the turnout main track is maintained in the main line. Due to the curving process conditions, in this case only a turnout with a radius of R_0 for which $R_{inn} \le R_0$ can be used (for $R_{inn} = R_0$ its diverging track will become a straight track).

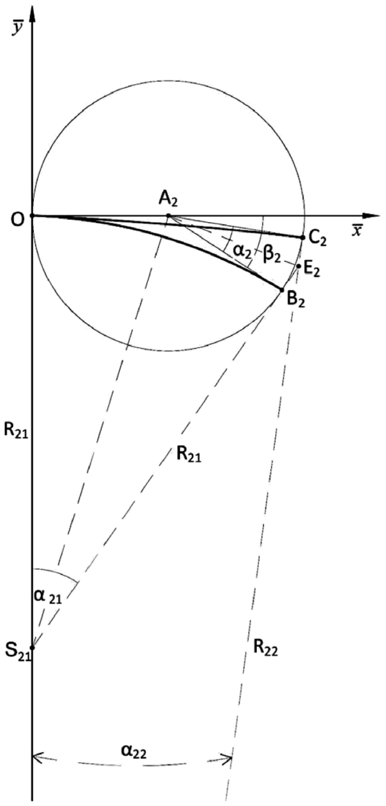

Fig. 2. Standard turnout scheme in variant III of curving: *O* – turnout toe, A_2 – standard turnout centre, B_2 – main track end, C_2 – diverging track end, S_{21} – main track curve centre [author's study]

The feature that distinguishes curved turnouts from typical turnouts is that they have the cant h_0 both in the main track and the diverging track. For this reason, the same rules apply for driving on the main turnout track as for driving on any track in a circular arc.

3. Determination of the radii of curved turnout diverging tracks

In order to determine vehicle speed for the main track connections, it is first necessary to specify the radii of the diverging tracks at the appropriate curved turnouts. The geometric situation is shown in Figure 3.

In the adopted system of rectangular coordinates, the beginning of a curved turnout in the outer track has an abscissa equal to zero, the end of its main track is at point B_1, and the end of the diverging track is at point C_1. The radius R_{12} of its diverging track must be determined. The beginning of a curved turnout in the inner track is located at the point *M*, the end of its main track at B_2, and the end of the diverging track at C_2. The radius R_{22} of its diverging track must be determined. The radii of the diverging tracks R_{12} (in variant I of curving) and R_{22} (in variant III of curving) are determined from the following formulas [13]:

Curved turnout in outer track

$$R_{12} = \frac{\overline{x}_{C1}^2 + \overline{y}_{C1}^2}{2\overline{y}_{C1}}, \tag{1}$$

$$\overline{x}_{C1} = t_1 + \frac{t_1}{\sqrt{1+\left[\tan(\alpha_1+\alpha_{11})\right]^2}}, \tag{2}$$

$$\overline{y}_{C1} = \frac{\tan(\alpha_1+\alpha_{11})}{\sqrt{1+\left[\tan(\alpha_1+\alpha_{11})\right]^2}}t_1, \tag{3}$$

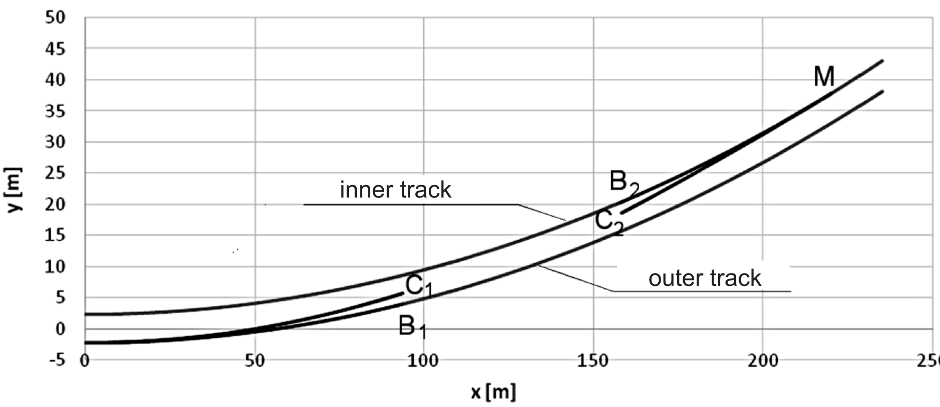

Fig. 3. Schematic geometry of parallel tracks with inserted curved turnouts (non-uniform scale): B_1 – end of the main turnout track in the outer track, C_1 – end of the turnout diverging track in the outer track, B_2 – end of the main turnout track in the inner track, C_2 – end of the diverging track in the inner track, *M* – toe of a turnout in the inner track [author's study]

$$t_1 = R_1 \tan\frac{\alpha_1}{2}, \tag{4}$$

$$\alpha_1 = \arctan\frac{1}{n_1}, \tag{5}$$

$$\alpha_{11} = 2\arctan\frac{t_1}{R_z}, \tag{6}$$

where:
- R_{out} – outer track curve radius [m],
- R_1 – diverging track curve radius at the standard turnout to be inserted in the outer track [m],
- R_{12} – diverging track curve radius at the curved turnout for the outer track [m],
- n_1 – a value specifying the bevel of the standard turnout to be inserted in the outer track,
- $\overline{x}_{C1}$ – the diverging track end abscissa in the outer track (in the turnout local system of coordinates) [m],
- $\overline{y}_{C1}$ – the diverging track end ordinate in the outer track (in the turnout local system of coordinates) [m],
- t_1 – tangent length of the standard turnout to be inserted in the outer track [m],
- α_1 – angle of the standard turnout to be inserted in the outer track [rad],
- α_{11} – the central angle of the main track curve in the curved turnout for the outer track [rad].

Curved turnout in inner track

$$R_{22} = -\frac{\overline{x}_{C2}^2 + \overline{y}_{C2}^2}{2\overline{y}_{C2}}, \tag{7}$$

$$\overline{x}_{C2} = t_2 + \frac{t_2}{\sqrt{1+\left[\tan(\alpha_2+\alpha_{21})\right]^2}}, \tag{8}$$

$$\overline{y}_{C2} = \frac{\tan(\alpha_2+\alpha_{21})}{\sqrt{1+\left[\tan(\alpha_2+\alpha_{21})\right]^2}} t_2, \tag{9}$$

$$t_2 = R_2 \tan\frac{\alpha_2}{2}, \tag{10}$$

$$\alpha_2 = \arctan\frac{1}{n_2}, \tag{11}$$

$$\alpha_{21} = 2\arctan\frac{t_2}{R_w}, \tag{12}$$

where:
- R_{inn} – inner track curve radius [m],
- R_2 – diverging track curve radius at the standard turnout to be inserted in the inner track [m],
- R_{22} – diverging track curve radius at the curved turnout for the inner track [m],
- n_2 – a value specifying the angle of the standard turnout to be inserted in the inner track,
- $\overline{x}_{C2}$ – the diverging track end abscissa in the inner track (in the turnout local system of coordinates) [m],
- $\overline{y}_{C2}$ – the diverging track end ordinate in the inner track (in the turnout local system of coordinates) [m],
- t_2 – tangent length of the standard turnout to be inserted in the inner track [m],
- α_2 – angle of the standard turnout to be inserted in the inner track [rad],
- α_{21} – central angle of the main track curve in the curved turnout for the inner track [rad].

4. Geometry selection for train speed analysis

Selected cases of geometry and the following basic formulas were used in the analysis:
- for a circular arc:

$$a_m = \frac{V^2}{(3,6)^2 R} - g\frac{h_0}{s} \le a_{dop}, \tag{13}$$

- for a transition curve (in the form of a clothoid):

$$\psi = \frac{a_m V}{3,6l} \le \psi_{dop}, \tag{14}$$

$$f = \frac{h_0 V}{3,6l} \le f_{dop}, \tag{15}$$

where:
- V – train speed [km/h],
- R – circular arc radius [m],
- h_0 – cant value on a circular arc [mm],
- s – track gauge (s = 1500 mm),
- g – standard gravity [m/s^2],
- a_m – unbalanced acceleration on a circular arc [m/s^2],

a_{per} – limit value for acceleration [m/s^2],
l – transition curve length [m],
ψ – acceleration change speed [m/s^3],
a_{per} – limit value of acceleration speed change [m/s^3],
f – rolling stock wheel lifting speed on the gradient due to cant [mm/s],
f_{per} – permitted rolling stock wheel lifting speed on the gradient due to cant [mm/s].

Table 1 presents a list of selected geometry cases obtained for admissible values of kinematic parameters: a_{per} = 0.85 m/s^2, ψ_{per} = 0.3 m/s^3, and f_{per} = 28 mm/s. The values given in Table 1 make it possible to estimate the train speed that will apply to the main track of a curved turnout after it has been laid in a circular arc. Of course, the number of existing possibilities is very high and the set speed can be achieved for different combinations of circular arc and cant radius. A simple rule applies here: the smaller the radius of the curve, the greater the cant value and the greater the length of the transition curve (gradient due to cant).

When considering this issue, it should be noted that the number of turnout types that can be curved is very limited. Table 2 summarises the values of the determined diverging track radii R_{12} and R_{22} for four types of standard turnouts: 1:26.5-2500, 1:18.5-1200, 1:14-760 and 1:12-500.

Table 2 shows that all the considered turnout types can be curved according to the direction of their diverging track (variant I of curving). The determined values R_{12} are, in each case, less than the corresponding values $R_{11} = R_{out}$ for their main track located in the outer track. At the same time, however, they increase in line with the radius of the standard turnout. Since the radius R_{12} most often determines the speed obtained at the track connection, it follows that curved turnouts should be used in the outer track, bent from the standard turnouts with the largest possible radius.

Table 2 also shows that the possibilities of using the considered turnouts in the inner track (in variant III of curving) are limited by the values of the turnout radius. Due to the track curvature conditions, the 1:26.5-2500 turnout can be used for R_{inn} ≤ 2500 m, whereby with R_{inn} = 2500 m its diverging track becomes a straight track. Similarly, the 1:18.5-1200 turnout can be used for R_{inn} ≤ 1200 m, the 1:12-760 turnout for R_{inn} ≤ 760 m, and the 1:12-500 turnout for R_{inn} ≤ 500 m.

Another conclusion resulting from Table 2 is that the turnout curving according to variant III (if it is intentional) in each case gives a larger diverging track radius than in variant I (with the same standard turnout radius). This means that the R_{22} radius does not determine the highest speed possible at a given track connection.

Table 1

List of selected geometry cases

Speed V [km/h]	**Curve radius R [m]**	**Cant h_0 [mm]**	**Acceleration a_m [m/s^2]**	**Transition curve length l [m]**
200	2500	70	0.777	144
	2000	110	0.824	219
160	1500	75	0.826	123
	1200	125	0.829	199
120	1000	60	0.719	80
	700	120	0.803	143
80	600	50	0.496	40
	400	100	0.581	80

[Author's study].

Table 2

The list of the values of the determined diverging track radii R_{12} and R_{22} for different radii R of the main lines

R_0 [m]	n	R [m]	R_{12} [m]	R_{22} [m]	R_0 [m]	n	R [m]	R_{12} [m]	R_{22} [m]
2500	26.5	3000	1363	–	1200	18.5	3000	857	–
		2500	1250	∞			2500	811	–
		2000	1111	10004			2000	750	–
		1800	1046	6432			1800	720	–
		1600	975	4447			1600	685	–
		1400	897	3184			1400	646	–
		1200	810	2309			1200	600	∞
		1000	714	1668			1000	545	6005
		900	661	1408			900	514	3604
		800	605	1178			800	479	2403
		700	546	973			700	442	1682
		600	483	791			600	399	1202
		500	416	626			500	352	859
		400	344	477			400	299	601
760	14	3000	606	–	500	12	3000	428	–
		2500	583	–			2500	417	–
		2000	550	–			2000	400	–
		1800	534	–			1800	391	–
		1600	515	–			1600	381	–
		1400	492	–			1400	368	–
		1200	465	–			1200	353	–
		1000	431	–			1000	333	–
		900	411	–			900	321	–
		800	389	–			800	307	–
		700	364	8883			700	291	–
		600	335	2856			600	272	–
		500	301	1465			500	250	∞
		400	261	847			400	222	2004

[Author's study].

5. Determination of train speeds on parallel track connections in a circular arc

Knowing the conditions presented, it is now possible to determine the achievable speed of trains on parallel track connections, the geometrical parameters of which are given in Table 1. The determined speeds are included in Table 3. The speed V_{div} results from the following formula, obtained from the formula rearrangement (13):

$$V_{zwr} = 3{,}6\sqrt{\left(a_{dop} + g\frac{h_0}{s}\right)R_{12}}\ . \qquad (16)$$

Table 3 shows that the train speed on parallel track connections depends on the type of standard curved turnout (i.e. mainly on its radius). The second very important factor is the value of the cant used. In any case, the set speed V_{div} is lower than the speed on the main tracks.

An interesting observation is the importance of the track cant here. As it turns out, the use of a standard curved turnout of a given type on main lines with a smaller radius and a correspondingly increased cant may have a better effect (i.e. a higher speed V_{div}) than in the case of a turnout on tracks with a larger radius and smaller cant. Thus, for the speed V = 200 km/h, a curved turnout created on the basis of a basic 1:26.5-

Table 3

Designated train speeds on parallel track connections

Speed V [km/h]	Curve radius R [m]	Cant h_0 [mm]	Turnout type	Curve radius R_{12} [m]	Speed V_{div} [km/h]
200	2500	70	1:26.5-2500	1250	145.555
			01:18.5-1200	811	117.242
			1:14-760	583	99.405
			01:12-500	417	84.070
	2000	110	1:26.5-2500	1111	150.323
			01:18.5-1200	750	123.509
			1:14-760	550	105.767
			01:12-500	400	90.198
160	1500	75	1:26.5-2500	937	127.587
			01:18.5-1200	667	107.646
			1:14-760	504	93.573
			01:12-500	375	80.715
	1200	125	1:26.5-2500	810	132.306
			01:18.5-1200	600	113.871
			1:14-760	465	100.245
			01:12-500	353	87.342
120	1000	60	1:26.5-2500	714	107.222
			01:18.5-1200	545	93.677
			1:14-760	431	83.305
			01:12-500	333	73.224
	700	120	1:26.5-2500	546	107.555
			01:18.5-1200	442	96.771
			1:14-760	364	87.818
			01:12-500	291	78.520
80	600	50	1:26.5-2500	483	85.835
			01:18.5-1200	399	78.015
			1:14-760	335	71.485
			01:12-500	272	64.413
	400	100	1:26.5-2500	344	81.885
			01:18.5-1200	299	76.342
			1:14-760	261	71.826
			01:12-500	222	65.781

[Author's study].

2500 turnout gives, on the geometry R = 2500 m, h_0 = 70 mm, the possibility of driving on a combination of parallel tracks at 145.555 km/h, while on the geometry R = 2000 m, h_0 = 110 mm it is 150.323 km/h. Of course, it must be borne in mind that the application of a larger cant means that longer (often significantly) transition curves must be introduced; in the first case, it will be l = 144 m, and in the second case l = 219 m. This may be a major problem in the case of a geometry for which an extension of the existing transition curves will have to be made.

The numerical values in Table 3 make it possible to assess the reduction in speed at the connections of parallel tracks in a curve. The smaller the radius of the standard turnout subjected to track curvature, the greater the decrease. In the calculations, with the speed on the main lines V = 200 km/h for the 1:26.5-2500 turnout it equals 25÷27%, for the 1:18.5-1200 turnout

– 38÷41%, for the 1:14-760 turnout – 47÷50%, and for the 1:12-500 turnout – 55÷58%. There is therefore no doubt that, in a given situation, the most advantageous would be to use the standard 1:26.5-2500 turnout.

For lower speeds on main tracks, the above trend is still valid, with increasingly lower speed drops. For the 1:26.5-2500 turnout and the speed V = 160 km/h the drop is 17÷20%, for V = 120 km/h – approx. 10%, and for V = 80 km/h there is no drop in speed. For the other considered turnouts, the speed drop is 29÷50% for V = 160 km/h, 19÷39% for V = 120 km/h and 3÷20% for V = 80 km/h.

The presented course of action, relating to the analysis of the train speed achieved, makes it possible to select the type of standard turnout that will be most advantageous for a given geometrical case.

6. Further speed limits resulting from track connection design rules

The general rules of connecting parallel tracks located in a circular arc – in an analytical notation – are presented in [13]. The idea of the connection is to insert two curved turnouts – in the outer track (variant I of curving) and in the inner track (variant III of curving). The effect obtained is shown in Figure 3, which shows that the key task during these operations is to connect the diverging tracks of both curved turnouts (i.e. points C_1 and C_2). The shape of this connection should correspond to the assumed kinematic conditions, ensuring smooth train passage, without adverse external impacts. In the first place, there should be no reverse curves.

In the adopted system of rectangular coordinates x, y, the position of the curved turnout in the outer track is clearly defined. This means that the coordinates of point C_1 and the tangent value at that point do not change during the whole process of determining the curvature connecting the diverging tracks. These values correspond directly to the effects of turnout curving, or more precisely to formulas (1)÷(3).

The key issue in this process is the correct location of the curved turnout in the inner track, i.e. finding the most favourable location for the toe of this turnout (i.e. point M). Theoretical relationships are given in [13], allowing the coordinates of the end of the diverging track (i.e. point C_2) and the tangent value at that point for any adopted point M to be determined.

In such a situation, the task is to connect point C_1 with the coordinates x_{C1}, y_{C1} and tangential gradient s_{C1} with point C_2 with the coordinates x_{C2}, y_{C2} and tangential gradient s_{C2}. The geometry shown in Figure 3 was based on the theoretical relationships from [13] and concerned two parallel track sections with the radii R_{out} = 702.295 m and R_{inn} = 697.705 m, with a 1:26.5-2500 curved turnout in the outer track and a 1:18.5-1200 turnout in the inner track. The values of R_{out} and R_{inn} radii result from the assumed circular arc radius running in the middle of the intertrack (R = 700 m) and the width of the intertrack spacing increased by the position of the route tracks in the curve (d = 4.59 m). The adopted coordinates for the toe of the curved turnout in the inner track are as follows: x_M = 220 m, y_M = 37.893 m.

The basic condition for proper connection of the curved turnout diverging tracks is associated with the relation between the values of the tangential gradients s_{C1} and s_{C2}. It should take the form $s_{C2} \geq s_{C1}$. If $s_{C2} < s_{C1}$ the inverse curvature appears in the geometry, which is not acceptable by definition. For $s_{C2} > s_{C1}$ the nature of the curvature occurring at the connection is consistent with the curvature of the turnout diverging tracks. At $s_{C2} = s_{C1}$ it is possible to connect the tracks by means of a straight insert if the ordinates y_{C1} and y_{C2} can be adjusted.

The analysis of effectiveness of track connecting methods on railway lines located in a circular arc in one paper [11] showed that the most advantageous solution is to connect the ends of diverging tracks using a circular arc. When using two curved standard turnouts of the same type, the required radius R_{13} of a circular arc that connects the ends of both diverging tracks is greater than the radius R_{12} and therefore does not require a train speed limit. On the other hand, when using a different type of standard turnout in the outer track than in the inner track, the required R_{13} appears to be smaller than the R_{12} radius; there is therefore a need for a further speed limit.

The straight tangent at the end of the curved turnout diverging track in the route track is also intended to be a straight tangent to an introduced circular arc of the radius R_{13}. The radius of this curve lies on the straight line of the equation:

$$y(x) = y_{C1} - \frac{1}{s_{C1}}\left(x - x_{C1}\right), \tag{17}$$

within R_{13} from point C_1. Hence, the coordinates of the curve centre:

$$x_{S13} = x_{C1} - \frac{s_{C1}}{\sqrt{1+s_{C1}^2}} R_{13}, \tag{18}$$

$$y_{S13} = y_{C1} + \frac{1}{\sqrt{1+s_{C1}^2}} R_{13}. \tag{19}$$

A circular arc is described by the formula:

$$y(x) = y_{S13} - \sqrt{R_{S13}^2 - \left(x - x_{S13}\right)^2}, \quad x \in \langle x_{C1}, x_{K13} \rangle. \tag{20}$$

If the tangent value for the circular arc end abscissa x_{K13}, still unknown in this stage, is s_{C2}, the formulas for the coordinates of point K_{13} follow:

$$x_{K13} = x_{C1} + \left(\frac{s_{C2}}{\sqrt{1+s_{C2}^2}} - \frac{s_{C1}}{\sqrt{1+s_{C1}^2}} \right) R_{13}, \quad (21)$$

$$y_{K13} = y_{C1} + \left(\frac{1}{\sqrt{1+s_{C1}^2}} - \frac{1}{\sqrt{1+s_{C2}^2}} \right) R_{13}. \quad (22)$$

In the presented solution, the connection of point K_{13} with point C_2 is made by means of a straight insert. However, as it turns out, the greater the radius value R_{13}, the shorter the section of this insert. This leads to the conclusion that the most advantageous solution, giving the largest radius R_{13}, would be to completely eliminate the straight insert and directly connect the ends of both diverging tracks by means of a circular arc (i.e. using a compound curve along the entire length of the connection). This would lead to two conditions being met: $x_{K13} = x_{C2}$ and $y_{K13} = y_{C2}$, on the basis of equations (21) and (22), two separate formulas for the radius R_{13} are derived:

$$R_{13} = \frac{\sqrt{1+s_{C1}^2}\sqrt{1+s_{C2}^2}}{s_{C2}\sqrt{1+s_{C1}^2} - s_{C1}\sqrt{1+s_{C2}^2}} \left(x_{C2} - x_{C1} \right), \quad (23)$$

$$R_{13} = \frac{\sqrt{1+s_{C1}^2}\sqrt{1+s_{C2}^2}}{\sqrt{1+s_{C2}^2} - \sqrt{1+s_{C1}^2}} \left(y_{C2} - y_{C1} \right). \quad (24)$$

So, in this situation, there is a general condition:

$$\frac{x_{C2} - x_{C1}}{s_{C2}\sqrt{1+s_{C1}^2} - s_{C1}\sqrt{1+s_{C2}^2}} - \frac{y_{C2} - y_{C1}}{\sqrt{1+s_{C2}^2} - \sqrt{1+s_{C1}^2}} = 0. \quad (25)$$

Condition (25) is met for a certain value x_M, as it determines the existing values x_{C2}, y_{C2} and s_{C2}, associated with the curved turnout in the inner track. The abscissa value x_M is determined in an iterative manner. Figure 4 shows an example of a connection of curved turnouts by using a circular arc in the case of the 1:18.5-1200 and 1:14-760 standard turnouts in the outer track and inner track, respectively.

Table 4 presents the results of the calculation of train speed on parallel track connections, carried out for a wide range of circular arc radii on the track, in the case of the standard curved turnouts of different types. This speed is limited by the value of the radius R_{13} and the cant h_0. It should be noted that for $R > 1200$ m the only way to solve the problem is to use two 1:26.5-2500 standard turnouts.

The data contained in Table 4 shows that the use of two different types of standard turnout leads to a further reduction in train speed – by several percent in relation to the speed resulting from the radius of the curve of the diverging track in the outer track.

7. Summary

Although the existing railway regulations introduce significant application restrictions for curved turnouts, this does not mean that they should be eliminated altogether. Since the use of these turnouts

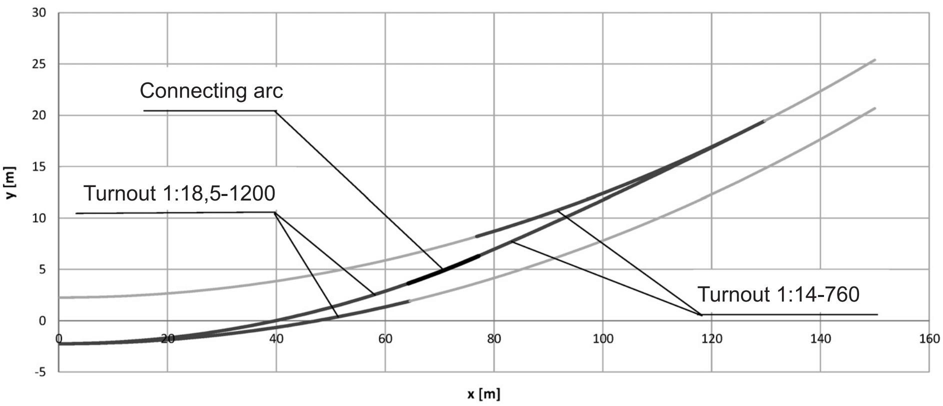

Fig. 4. View of the connection of curved turnouts through the use of a circular arc (n_1 = 18.5, R_1 = 1200 m, R_{out} = 502.25 m, R_{12} = 353.444 m, n_2 = 14, R_2 = 760 m, R_{inn} = 497.75 m, x_M = 129.759 m, R_{13} = 303.766 m; on a non-uniform scale) [author's study]

Table 4

Train speeds when connecting tracks in a curve by using a compound curve when using different types of standard turnout

V [km/h]	R [m]	h_0 [mm]	n_1	R_1 [m]	R_{12} [m]	V_{div} [km/h]	n_2	R_2 [m]	R_{13} [m]	V_{13} [km/h]
160	1200	125	26.5	2500	811.232	132.232	18.5	1200	618.493	115.612
	1000	60	26.5	2500	714.780	107.280	18.5	1200	547.557	93.896
120	900	80	26.5	2500	662.276	108.565	18.5	1200	485.829	92.985
	800	100	26.5	2500	606.643	108.741	18.5	1200	472.016	95.919
	700	120	26.5	2500	547.550	107.708	18.5	1200	455.196	98.205
	800	50	26.5	2500	606.643	96.196	18.5	1200	472.016	84.853
100	700	70	26.5	2500	547.550	96.335	18.5	1200	455.196	87.836
	600	90	18.5	1200	400.387	86.400	14	760	309.760	75.995
	500	120	18.5	1200	353.444	86.536	14	760	303.765	80.224
80	600	50	18.5	1200	400.387	78.150	14	760	309.760	68.739
	500	70	18.5	1200	353.444	77.399	14	760	303.765	71.753

[Author's study].

on railroads is allowed in economically justified cases, there is sufficient argument to use them. As it should be assumed, the fears resulting from the lack of proper knowledge and attempts to avoid responsibility play an important role in all this. The analytical approach presented in this article should dispel these concerns to a large extent.

The general rules of connecting parallel tracks located in a circular arc (in an analytical notation) are presented in one paper [13]. The connection idea is to insert two curved turnouts: one curved in the outer track in the direction of its diverging track and in the inner track in the opposite direction to the diverging track. The key task during these operations is to connect the diverging tracks of both curved turnouts with each other. The shape of this connection should correspond to the assumed kinematic conditions, ensuring smooth train passage, without adverse external impacts. In the first place, there should be no reverse curves.

The need to use curved turnouts, which occurs when parallel tracks in a circular arc are connected, leads to a reduction in a train's achievable speed. The value of this limit is, in most cases, determined by the diverging track curve radius of the curved turnout located in the outer route track. The value of this radius results from the radius of a circular arc occurring in the main track and the type of the standard turnout adopted for curving. For this reason, it is most advantageous to use the standard turnout with the largest possible radius in the diverging track for curving.

The analysis of effectiveness of track connecting methods on railway lines located in a circular arc in [11] showed that the most advantageous solution is to connect the ends of diverging tracks using a circular arc (without a straight insert). When using two basic curved turnouts of the same type, the required radius of a circular arc that connects the ends of both diverging tracks is greater than the radius of the diverging track curve in the outer route track and, therefore, does not necessitate a further speed limit.

When using a different type of standard turnout in the outer track than in the inner track, the required radius of the curve connecting the ends of both diverging tracks is smaller than the diverging track curve radius in the outer track, which leads to a further reduction by a dozen or more percent of the train speed. This way of connecting the tracks should not be recommended in this case.

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