However, nothing prevents the use of other, intermediate l_c values as well, following the suggestions mentioned above, as this will most often not have a significant effect on the accuracy of the achieved curvature of the track axis. In this respect, the moving chord method is very flexible.

It is also possible to adopt a different (i.e. shorter) chord when determining the curvature along the length of the transition curve than is the case for a circular curve. During the analysis for the test section V = 80 km/h, a chord of l_c = 30 m would be most favourable for the identification of the curvature of the circular curve and a chord of l_c = 20 m for the identification of the curvature of the transition curve. For the test section V = 120 km/h, a chord l_c = 40 m for the circular curve and l_c = 30 m for the transition curves would be suitable, while the appropriateness of using a chord l_c = 40 m for the identification of transition curves on the test section V = 120 km/h should be considered (the curvature of the circular curve must be determined here using a chord l_c = 50 m).

7. Conclusions

The basis for determining the position of straight sections of the railway track and sections located in a curve, as well as for determining the corresponding geometric parameters, is the knowledge of the curvature of the track axis. The works [23–25] present the concept of a new method for determining curvature (called the "moving chord method") and its verification on an unambiguously defined elementary geometric layout of tracks.

The papers [26, 30, 31] discuss the procedure for estimating the curvature of a track axis by the moving chord method using Cartesian coordinates obtained by direct measurements carried out. It was shown that values of circular curve radius and lengths of transition curves, as well as the location of so-called segmentation points (i.e. boundary points between straight sections, transition curves and circular curves) can be determined from the curvature diagram.

This article focuses on the issue of selecting the chord length that will be most beneficial in a particular situation Although for the model systems, as shown in the work [24], the effect of the applied chord length on the determined curvature values turned out to be insignificant, in the operated railway track, due to track deformations and measurement error, the situation may differ significantly. This is evidenced, for example, by the irregular curvature diagrams shown in the paper [31]. The study analysed three test geometric layouts adapted to the speeds of 80 km/h, 120 km/h and 160 km/h (the radii of circular arcs determined as a result of the curvature estimation performed were approximately 410 m, 880 m and 1480 m, respectively). The lengths of the moving chord in the range of 10÷50 m were considered.

On the basis of the conducted analysis, it was unequivocally demonstrated that the chord length used to determine the curvature in the railway track in service should depend on the value of the radius of the circular curve. The virtual chord should not be too short; for example, using a chord length of l_c = 10 m is not a reasonable solution. A chord l_c = 20 m may also be of little use for large circular curve radii; in such cases, a chord l_c = 50 m should be considered. Indicative lengths of l_c have been suggested, depending on the range of values of R_{CA}, but nothing prevents the use of other, intermediate values of l_c, as this will usually not have a significant impact on the accuracy of the achieved curvature of the track axis. In this respect, the moving chord method is very flexible. It is also possible to adopt a different (i.e. slightly shorter) chord when determining the curvature along the length of the transition curve than is the case for a circular curve. Often, the position and length of the transition curve can be determined more accurately from the curvature diagram obtained using a shorter chord.

The issues presented in the article, concerning the adaptation of the moving chord method to the adopted measurement procedure and the use of the obtained curvature diagram, provide a suitable application basis for the described method. Implementation of the presented procedure should significantly improve the process of identification of geometric layouts of the track in the horizontal plane.

References

1. Deutsche Bahn: *883.2000 DB_REF-Festpunktfeld*, Deutsche Bahn Netz AG, Berlin, Germany, 2016.
2. European Committee for Standardization (CEN): *Railway applications – Track–Track alignment design parameters – Track gauges 1435 mm and wider. Part 1: Plain line. EN 13803-1*, Brussels, Belgium, 2010.
3. Federal Railroad Administration: *Code of federal regulations title 49 transportation*, US Government Printing Office, Washington, DC, 2008.
4. Network Rail: *NR/L3/TRK/0030 NR Reinstatement of Absolute Track Geometry (WCRL Routes), Iss. 1*, London, UK, 2008.
5. New South Wales: *Standard: Railway Surveying, Version 1.0. T HR TR 13000 ST*, Government (Transport for NSW), Sydney, Australia, 2016.
6. Österreichische Bundesbahnen: *Linienführung von Gleisen, B 50 – Oberbau – Technische Grundsätze. Teil 2*, GB Fahrweg Technik, Wien, Austria, 2004.
7. Standardy Techniczne – Szczegółowe warunki techniczne dla modernizacji lub budowy linii kolejowych

do prędkości V_{max} ≤ 200 km/h (dla taboru konwencjonalnego) / 250 km/h (dla taboru z wychylnym pudłem) – TOM I – DROGA SZYNOWA – Załącznik ST-T1-A6: Układy geometryczne torów, PKP Polskie Linie Kolejowe S.A., Warsaw, 2018.
8. Schweizerische Bundesbahnen: *Ausführungsbestimmungen zur Eisenbahnver-ordnung. SR 742.141.11*, Ministerium für Verkehr, Bern, Switzerland, 2016.
9. Szwilski A.B. et.al.: *Employing HADGPS to survey track and monitor movement at curves*, In Proc. 8th Int. Conf. "Railway Engineering 2005", London, UK, Engineering Technics Press, Edinburgh.
10. Li W. et.al.: *A method for automatically recreating the horizontal alignment geometry of existing railways*, Computer Aided Civil and Infrastructure Engineerig, vol. 34, iss. 1/2019. pp. 71–94, Wiley Online Library.
11. Pu H. et.al.: *A global iterations method for recreating railway vertical alignment considering multiple constraints*, IEEE Access, vol. 7, iss. 1/2019, pp. 121199–121211, Institute of Electrical and Electronics Engineers.
12. *A guide to using IMU (accelerometer and gyroscope devices) in embedded applications*, Starlino Electronics, 2009, Web side: http://www.starlino.com/imu_guide.html.
13. Guimarães-Steinicke C. et.al.: *Chapter Four – Terrestrial laser scanning reveals temporal changes in biodiversity mechanisms driving grassland productivity*, Advances in Ecological Research, vol. 61, 2019, pp. 133–161, Academic Press.
14. Alkan R.M.: *Cm-level high accurate point positioning with satellite-based GNSS correction service in dynamic applications*, Journal of Spatial Science, vol. 66, iss. 2/2019, pp. 351-359, Taylor & Francis.
15. Chang, L. et.al.: *Railway infrastructure classification and instability identification using Sentinel-1 SAR and Laser Scanning data*, Sensors, vol. 20, iss. 24/2020, 7108, Multidisciplinary Digital Publishing Institute.
16. Quan, Y., Lau L.: *Development of a trajectory constrained rotating arm rig for testing GNSS kinematic positioning*, Measurement, vol. 140, 2019, pp. 479–485, Elsevier.
17. Wang, L. et.al.: *Validation and assessment of multi-GNSS real-time precise point positioning in simulated kinematic mode using IGS real-time service*, Remote Sensing, vol. 10, iss. 2/2018, 337, Multidisciplinary Digital Publishing Institute.
18. Wu S. et.al.: *Improving ambiguity resolution success rate in the joint solution of GNSS-based attitude determination and relative positioning with multivariate constraints*, GPS Solution, vol. 24, iss. 1/2020, 31, Springer.
19. Koc W.: *Design of rail-track geometric systems by satellite measurement*, Journal of Transportation Engineering, vol. 138, iss. 1/2012, pp. 114–122, American Society of Civil Engineers.
20. Koc W.: *Analytical method of modelling the geometric system of communication route,* Mathematical Problems in Engineering, vol. 2014, 679817, Hindawi Publishing Corporation.
21. Koc W.: *Design of compound curves adapted to the satellite measurements*, The Archives of Transport, vol. 34, iss. 2/2015, pp. 37–49, Polska Akademia Nauk, Komitet Transportu.
22. Koc W.: *Design of reverse curves adapted to the satellite measurements, Advances in Civil Engineering*, vol. 2016, 6503962, Hindawi Publishing Corporation.
23. Koc W.: *The method of determining horizontal curvature in geometrical layouts of railway track with the use of moving chord*, Archives of Civil Engineering, vol. 66, iss. 4/2020, pp. 579–591, Warsaw University of Technology, Faculty of Civil Engineering.
24. Koc W.: *Analiza skuteczności wyznaczania poziomej krzywizny osi toru z wykorzystaniem ruchomej cięciwy* [Analysis of the effectiveness of determining the horizontal curvature of a track axis using a moving chord], Problemy Kolejnictwa, 2021, iss. 190, pp. 77–86.
25. Koc W.: *Analysis of moving chord inclination angles when determining curvature of track axis*, Current Journal of Applied Science and Technology, vol. 40, iss. 10/2021, pp. 92-103, Article no. CJAST.68309, SCIENDOMAIN International.
26. Koc W.: *Estimation of the horizontal curvature of the railway track axis with the use of a moving chord based on geodetic measurements*, Journal of Surveying Engineering, vol. 148, iss. 4/2022, 04022007, American Society of Civil Engineers.
27. Regulation of the Council of Ministers of 15 October 2012 on the national spatial reference system, Dz.U. /Journal of Laws/ of 2012, item 1247.
28. Moritz H.: *Geodetic Reference System 1980*, Bulletin Géodésique, vol. 54, iss. 3/1980, pp. 395–405, Springer Link.
29. Turiño C.E.: *Gauss Krüger projection for areas of wide longitudinal extent*, International Journal of Geographical Information Science, vol. 22, iss. 6/2008, pp. 703–719, Taylor & Francis Online.
30. Koc W.: *Identification of geometrical parameters of an operational railway route determined by the curvature of the track axis*, European Journal of Applied Sciences, vol. 10, iss. 5/2022, pp. 129–148, Services for Science and Education, United Kingdom.
31. Koc W. et.al.: *Determining horizontal curvature of railway track axis in mobile satellite measurements*, Bulletin of Polish Academy of Sciences: Technical Sciences, iss. 6/2021, e139204, Polska Akademia Nauk.