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Determination of directional angle of a railway route and curvature of the railway track axis on high-speed railways

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Summary

The paper describes the assumptions underlying two new calculation methods: the determination of the directional angle of a railway route and the determination of the curvature of a railway track axis. These methods use measurement data in the form of Cartesian coordinates of the track axis (obtained during the survey), while the basis for the calculation is the identification of a virtual chord projected in a horizontal plane, which connects two points on the track axis. The key role for the calculation is played here by the determined slope of the tangent to the track axis. Subsequently, an attempt was made to test the extent to which these methods correspond to the conditions specific for high-speed railways. This was illustrated using two calculation examples involving geometric layouts for speeds of 260 km/h and 350 km/h. In order to keep the applicability to reality to a greater extent, a decision was made to obtain hypothetical measurement data by virtual modification of these layouts. Qualitatively, the results of the analysis carried out were no different from previous analyses relating to standard railways. In particular, they confirmed beyond doubt the suitability of the methods considered for determining the directional angle of the railway route and determining the curvature of the railway track axis on high-speed railways. As shown, for use on these railways, a chord length of l_c = 100 m should be recommended.

Keywords: railway, high-speed rail, directional angle of a railway route, curvature of the railway track axis, calculation algorithms, example geometric layouts

1. Introduction

As we celebrate the 100th anniversary of railway research in Poland, it is important not to overlook the fundamental limitation of not having a functioning high-speed railway system. These railways set the world level of technology in the field of railways and, moreover, as it is commonly believed, indicate a high level of civilisation. Looking from the perspective of other countries (with which we identify), the described underdevelopment has been going on in Poland for several decades, and the possibilities of changing this situation are still not fully defined. This also has an impact on the ongoing research topic due to the lack of a key research object.

This obviously does not mean that research work useful from the point of view of high-speed rail cannot be carried out in Poland. Railway engineering is such a broad field that it is possible to distinguish fields which are of universal character and concern not only standard railways. As it seems, this concerns in particular the principles of designing track geometric layouts and the broadly understood maintenance diagnostics of these layouts.

Regardless of type of railway, whether standard or high-speed, the shape of the existing railway track gradually changes over many years of use. After a certain period of time, it starts to deviate from its original layout, designed in accordance with the regulations in force. Particularly disadvantageous can be deviations of the track axis in the horizontal plane. Therefore, it is necessary to know the current geometrical shape of the railway route so that anomalies can be eliminated and possible corrections can be made. This is done with the support of appropriate measurement systems and calculation algorithms.

The measurement methods currently in use are similar across different railway administrations [4, 5, 8, 24, 32]. In addition to classical surveying methods, geostationary satellite measurements based on Global Navigation Satellite System (GNSS) are used, which utilise so-called active geodetic networks. Mobile sat-

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ellite surveying methods are also being introduced [31], in which, in addition to GNSS receivers, Inertial Navigation System (INS) devices [1] and visual methods such as Terrestrial Laser Scanning (TLS) [10] are often used for support. The possibility of using systems consisting of satellite receivers mounted on different types of vehicles is being investigated [26, 34].

Measurements carried out provide the basis for the identification of a railway route. It consists in determining its geometrical parameters by using the determined Cartesian coordinates of the railway track axis points in the appropriate state spatial reference system. For example, in Poland – with regard to plane coordinates – the PL-2000 system is in force [27]. This system was created by assigning points on the GRS 80 reference ellipsoid [23] to corresponding points on the plane, according to the Gauss-Krüger projection [33]. From a mathematical point of view, this assignment is unambiguous.

The current approach to the problem of identifying a measured railway route is to generate an optimised geometric layout in a given area by minimising the deviations of this layout from the measurement points (while meeting the relevant maintenance and operational requirements). The standard solution to a design problem relies heavily on human experience [18, 35]. Many works attempt to automate this process. There are two variants: geometric identification [9, 19, 22, 25] and alignment optimisation [3, 6, 7, 28–30]. The works [2, 20, 21] propose several methods for combining these variants into an iterative process of railway track axis reconstruction. There is a trend that most of the works mentioned in recent years have been developed by Chinese researchers, so they are undoubtedly relevant to the conditions specific for high-speed railways.

As it turns out, the problem of geometric identification of a railway track can be approached in a completely different way, by moving away from fitting a hypothetical model system to the railway track axis points determined by direct measurements. The basis of identification can be the curvature of the geometric layout; in this case, it would only be necessary to develop a suitable method for its determination. The works [12, 15, 16] propose a new method of determining the curvature of the railway track axis, referred to as the "moving chord method".

However, in each case, the identification of the geometric layout of the railway track is based on Cartesian coordinates determined by field measurements. Most often, these are measurements using a ground-based geodetic network matrix, although GNSS measurements, conducted in a static or mobile manner, are becoming increasingly popular. Mobile satellite measurements appear to be the most effective method for determining the coordinates of a railway track axis. This method is by far the fastest and least demanding, and provides an incomparable amount of measurement data. The fact that this method is not widely used is probably due to the issue of its accuracy, which has not been fully investigated.

However, there are situations when the satellite signal can be disturbed (by the occurring terrain obstructions) or completely disappear (e.g., in tunnels). In such cases, the ability to measure the value of the directional angle of the railway route Φ using an inertial system is of great help. Knowledge of this angle, along with simultaneous measurement of the distance ΔL, allows to determine the Cartesian coordinates of the next measurement point (knowing the coordinates of the previous point). For the PL-2000 system, the following formulae apply:

$$Y_{i+1} = Y_i + \Delta L \cdot \cos \Phi_{i \div i+1} \tag{1}$$

$$X_{i+1} = X_i + \Delta L \cdot \sin \Phi_{i \div i+1} \tag{2}$$

In this situation, the directional angle of the railway route can also be used in the process of the geometric identifying of the railroad track axis.

This article attempts to investigate the extent to which high-speed railroads are compatible with recently developed methods for determining the directional angle of the railway route and curvature of the railway track axis. The methods are based on two basic principles:

- measurement data in the form of Cartesian coordinates of the railway track axes obtained during the survey is used,
- the basis of the calculation is the identification of a virtual chord exposed in the horizontal plane that connects two points on the railway track axis.

2. Method for determining the directional angle of a railway route

2.1. Definition of directional angle

The concept of directional angle is particularly relevant in the field of traditional maritime and aerial navigation. In maritime navigation there is a term for the so-called centreline, which is an imaginary line connecting the bow and stern of a vessel, located in the plane of symmetry of the vessel. The centreline has no specific height in this plane, but is parallel to the horizon. This is directly related to the term course, which is the (horizontal) direction in which the vessel is steered or should be steered. The course is determined as a measure of the angle (in degrees, clockwise) from the north reference direction. The *true* *course* (TC) which is of great interest to us is the angle between true north (meridian) and the front of the ship's centreline.

One can easily see that the concept of the centreline can also be applied to the longitudinal axis of a railway carriage, determined by the line joining the bogie or wheelset pivots. The angle of inclination of the centreline with respect to the north direction can therefore be regarded as the directional angle (i.e. course) of the rail vehicle. In the straight sections of the route, the centreline of the carriage coincides with the axis of the railway track, and in curves it is parallel to the straight line tangent to the railway track axis. The directional angle of a rail vehicle is closely related to the direction of its movement on a given railway line (it determines the course of the carriage expressed in degrees).

The directional angle of a route is determined on a topographic map as the angle between the direction of inclination of the tangent to the geometric layout and the reference direction, which is a northerly direction. The values of the directional angle at a given point of the railway route are derived from the angle of inclination of the tangent to the railway track axis at that point. On straight sections, this tangent coincides with the railway track axis, so it can be easily determined by surveying, specifying the relevant Cartesian coordinates [17] On curved sections (i.e. transition curves and circular curves), the matter is already more complicated, as direct determination of tangents is difficult in this case.

It should be further noted that the directional angle of a rail vehicle is closely related to the direction of its movement on a given railway line, whereas a railway line does not have such a defined direction. Therefore, such a direction should be adopted, bearing in mind that it will be symbolic in nature. As it seems, in order to keep the directional angle of the railway route unambiguous, it will be most beneficial to match the increasing kilometers of the railway line.

In any case, there is a need to develop an effective method for determining both the directional angle of the railway line and the directional angle of the rail vehicle, based on appropriate measurement data. The latter should be the Cartesian coordinates of the railway track axis, allowing visualisation of the given railway route and obtaining a general information about its course. The presented proposal for solving the posed problem refers to the assumptions made as part of a new method of determining the curvature of a railway track axis using a moving chord [12, 15, 16].

2.2. Basic relations

If one denotes by $_i$ the angle of inclination of the tangent to the railway track axis in the rectangular coordinate system, the directional angle of the railway route Φ_i at a given point *i* is directly derived from the value of this angle. To find it, a simple transformation is needed, involving expressing the angle $_i$ in degrees and applying the appropriate formula. For $\Theta_i \in \langle 0;90^\circ \rangle$, $\Theta_i \in \langle 0;-90^\circ \rangle$ and $\Theta_i \in \langle -90^\circ;-180^\circ \rangle$, the following formula applies:

$$\Phi_i = 90^\circ - \Theta_i \tag{3}$$

while for $\Theta_i \in \langle 90^\circ;180^\circ \rangle$

$$\Phi_i = 360^\circ + (90^\circ - \Theta_i) \tag{4}.$$

2.3. Assumptions

Therefore, the basic problem to be solved remains the determination of the value Θ_i of the tangent angle at a given point. For this purpose, a chord of a given length stretched along the railway track can be used, as is the case when determining the curvature of the railway track axis by the moving chord method [12, 15, 16].

When using this method, the primary task of the procedure for determining the curvature at the measuring point *i* – based on the determined Cartesian coordinates – is to determine the values of the angles of the two virtual chords (derived from this point forward and backward) to the axis of abscissas of the corresponding rectangular coordinate system. This is shown in Figure 1 (for a segment of a circular curve).

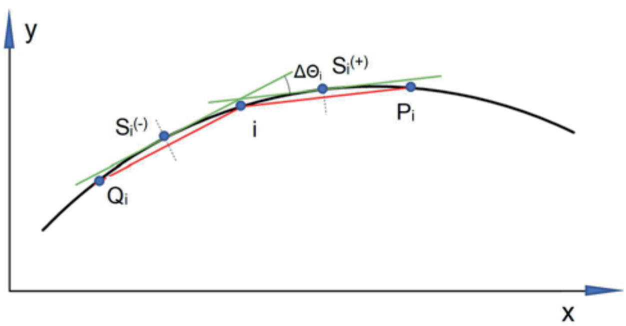

Fig. 1. Schematic diagram of the determination of moving chord angles [author's own elaboration]

The method in question employs two key assumptions:

(*1*) ***The lines tangent to the railway track axis and the corresponding chord are parallel to each other***;

(*2*) ***The points of tangency project perpendicularly to the centre of the given chord.***

These assumptions are met for the circular curve. However, along the length of the transition curve, this is no longer the case, with the inconsistency resulting from a strict failure to meet these conditions being relatively small; this decreases as one moves into the initial region of the transition curve.

The angles of both chords (front $\Theta_i^{(+)}$ and back $\Theta_i^{(-)}$) do not refer to a given measurement point, but to points distant from point *i* by the value corresponding to half the chord length. To create the $\Theta(L)$ relationship graphs, it would be necessary to determine the linear coordinates of these points separately for each chord, which would require an additional calculation. However, as it turns out, there is no need for this, as the hypothetical angle ${}_i$ of the tangent at point *i* can be determined in another, much simpler way, using the values of the angles of the two virtual chords.

Assuming the determined values of the $\Theta_i^{(+)}$ and $\Theta_i^{(-)}$ angles as the basis for the calculations, the angle ${}_i$ of the tangent at point *i* was assumed to be the arithmetic mean value of these angles.

$$\Theta_i = \frac{\Theta_i^{(+)} + \Theta_i^{(-)}}{2} \tag{5}$$

2.4. Calculation algorithm

The methodology for determining the values of the $\Theta_i^{(+)}$ and $\Theta_i^{(-)}$ angles at any measurement point is explained in detail in the works [13, 16] dealing with the method of determining the curvature of the railway track axis. The sequence of operations carried out for this purpose is illustrated in Figure 2.

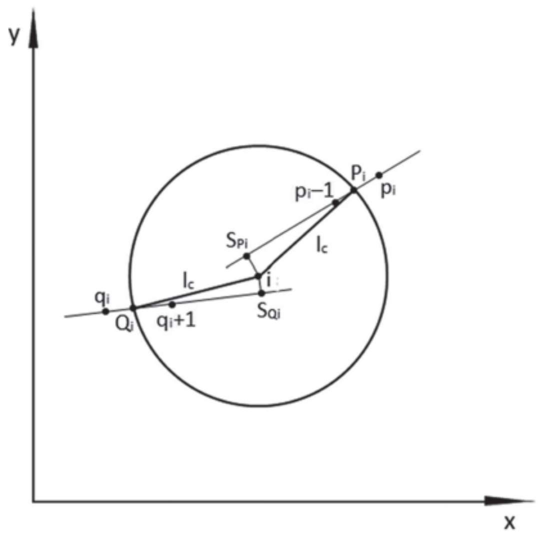

Fig. 2. Explanation of the method of obtaining data for determining the values of the virtual chord angles derived from point *i* [author's own elaboration]

The procedure the data for determining the values of angles $\Theta_i^{(+)}$ and $\Theta_i^{(-)}$ begins with measurement point *i*, which is located in such a way as to allow a virtual chord of length l_c to be projected backwards; the end of the calculation must take place at a point from which a virtual chord of the same length can still be projected forwards. The basic operation that must first be carried out is to determine the numbering of the points determining the intervals in which the ends of the virtual chords derived from point *i* are located.

In the case of a chord derived from point *i* forward, the interval in which the end of the chord occurs is determined by the points p_i–1 and p_i (Fig. 1). Once these have been determined, it is possible to provide an analytical description of the equation of the straight line passing through these points. This equation has the following general form:

$$y = a_{pi} + b_{pi} x \,. \tag{6}$$

As shown in Figure 1, the projected end of the front chord (i.e. point P_i) lies on the straight line described by equation (6), at a distance l_c from point *i*. It thus represents the point of intersection of a circle of radius l_c and centre at point *i* with the line (6). The coordinates of point P_i are determined from the following formulae:

$$x_{Pi} = \frac{-B_{Pi} \pm \sqrt{B_{Pi}^2 - 4A_{Pi}C_{Pi}}}{2A_{Pi}} \tag{7}$$

$$y_{Pi} = a_{pi} + b_{pi} \frac{-B_{Pi} \pm \sqrt{B_{Pi}^2 - 4A_{Pi}C_{Pi}}}{2A_{Pi}} \tag{8}$$

where:

$$A_{Pi} = 1 + b_{pi}^2$$

$$B_{Pi} = -2\left(x_{Spi} + b_{pi} y_{Spi} - a_{pi} b_{pi}\right)$$

$$C_{Pi} = x_{Spi}^2 + y_{Spi}^2 - 2a_{pi} y_{Spi} + a_{pi}^2 - l_c^2 + \left[\left(x_i - x_{Spi}\right)^2 + \left(y_i - y_{Spi}\right)^2\right]$$

$$x_{Spi} = \frac{b_{pi}}{1 + b_{pi}^2}\left(y_i + \frac{1}{b_{pi}} x_i - a_{pi}\right)$$

$$y_{Spi} = \frac{1}{1 + b_{pi}^2}\left(b_{pi}^2 y_i + b_{pi} x_i + a_{pi}\right).$$

The “+” sign in formulas (7) and (8) occurs when the values of the abscissae of the measured points of the route are increasing, while the “–” sign is valid for decreasing abscissae.

For a chord derived from point i backwards, the interval in which the end of the chord occurs is determined by points q_i and q_i+1 (Fig. 1). It is determined in a similar way to that which was done for the chord moving forward. The further steps are also similar and lead to the determination of the coordinates of point Q_i.

With the Cartesian coordinates of point i (obtained from measurements) and the coordinates of the ends of the forward and backward virtual chords, the values of the $\Theta_i^{(+)}$ and $\Theta_i^{(-)}$ angles of these chords can be determined, followed by the value of the tangent angle at the given measurement point. A chord moving forward connects point i with point P_i, whose coordinates are determined using formulae (7) and (8). The angle of inclination is:

$$\Theta_{i \div Pi} = \Theta_i^{(+)} = \arctan \frac{y_{Pi} - y_i}{x_{Pi} - x_i} \quad (9)$$

A chord moving backwards connects the i point with the Q_i point. The angle of inclination is:

$$\Theta_{Qi \div i} = \Theta_i^{(-)} = \arctan \frac{y_i - y_{Qi}}{x_i - x_{Qi}} \quad (10)$$

In this situation, the value of the angle of inclination of the tangent at a given measurement point is determined using formula (5). The presented course of action is sequential and consists in the use of the given calculation formulas. The determination of the value of the directional angle does not require the development of special computer programs, and the whole operation can be carried out, for example, in a spreadsheet.

A detailed analysis carried out in the work [11] clearly confirms the very high precision of the proposed method of determining the directional angle of the railway route and the lack of significance of the assumed length of the virtual chord.

3. Method of determination of the curvature of a railway track axis

Taking into account the definition of curvature resulting from formula

$$\kappa = \lim_{\Delta l \to 0} \left| \frac{\Delta\Theta}{\Delta l} \right| = \frac{d\Theta}{dl} \quad (11)$$

for the situation shown in Figure 1 the following formula can be developed for practical use:

$$\tilde{\kappa}_i = \frac{\Delta\Theta_i}{\overset{\frown}{S_i^{(-)} S_i^{(+)}}} \quad (12)$$

In formula (12), $\Delta\Theta_i = \Theta_i^{(+)} - \Theta_i^{(-)}$ represents the difference in angles of the tangents to the geometric layout displayed at points $S_i^{(-)}$ and $S_i^{(+)}$, and $\overset{\frown}{S_i^{(-)} S_i^{(+)}}$ is the distance between these points .

While knowledge of the Cartesian coordinates makes it easy to determine the $\Delta\Theta_i$ angle difference, this is not the case for determining the $\overset{\frown}{S_i^{(-)} S_i^{(+)}}$ distance. In the case of a circular curve, it can be determined from the formula

$$\overset{\frown}{S_i^{(-)} S_i^{(+)}} = 2 \cdot R \cdot \arcsin \frac{l_c}{2R}, \quad (13)$$

however, from a practical point of view, this formula is of little use, as the value of the curve radius R is not known. In such a situation, the use of the moving chord method requires adopting a third additional assumption in addition to those given in subsection 2.3:

(*3*) ***curvature κ_i occurring at a given point i is determined by the following formula***:

$$\kappa_i = \frac{\Delta\Theta_i}{l_c} \quad (14)$$

It goes without saying that in the case of curved sections, the values of curvature obtained using formulae (12) and (14) must differ, since the condition $\overset{\frown}{S_i^{(-)} S_i^{(+)}} > l_c$ applies in each case. Therefore, assumption (*3*) implies a limitation of the range of applicability of the method of determining curvature; the assumptions adopted therein generally apply only to a certain, limited range of geometrical parameters.

In order to determine the scope of applicability of the moving chord method, a numerical indicator

$$d = \frac{l_c}{\overset{\frown}{S_i^{(-)} S_i^{(+)}}} \quad (15)$$

was introduced to compare the values occurring in the denominator of formulas (12) and (14) with each other. This was possible for a circular curve, where the distance $\overset{\frown}{S_i^{(-)} S_i^{(+)}}$ was derived from formula (13). Figure 3 shows graphs of the d index calculated using formula (15) for circular curve radii in the

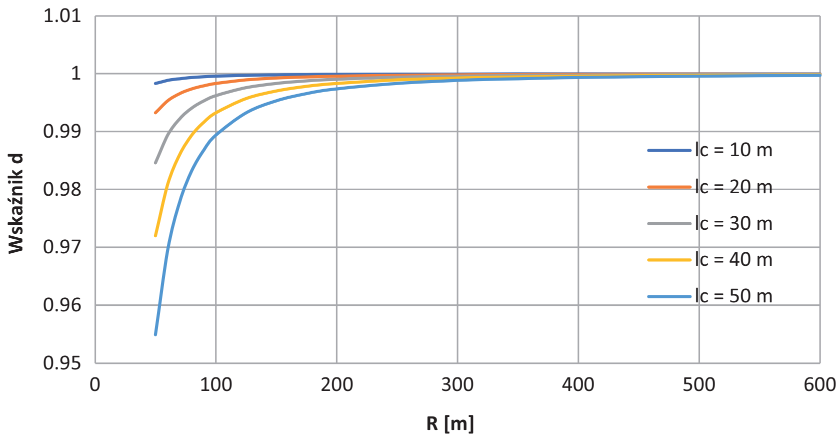

Fig. 3. Relationship graphs $d(R)$ for different values of l_c (at $R = 50 \div 600$ m) [author's own elaboration]

range $R = 50 \div 600$ m, with moving chord lengths $l_c = 10 \div 50$ m.

As one can see, the values of the adopted d index decreases (i.e. the more it differs from unit), the smaller the radius of the circular curve and the greater the length of the chord used. At the same time, the graphs in Figure 3 fully clarify the question of the range of applicability of the described method. As it turns out, the moving chord method fully corresponds to the conditions found on railway routes. Here, circular curve radii $R \leq 300$ m are basically only used in turnouts, possibly on mountain railways. In typical situations, they are considerably larger so that, depending on the category of the line in question, the speed of the trains is not restricted.

Figure 4 shows graphs of the relationship $d(R)$ for a typical range of radii found on railways, taking into account situations arising on high-speed railways. At first glance, this Figure is similar to Figure 3, but, as it turns out, the values on the ordinate axis here are one order of magnitude smaller. This means that in practice the $\widehat{S_i^{(-)}S_i^{(+)}}$ lengths measured along the curve are almost the same as the length of the corresponding chord. For high-speed railways (i.e. for $R > 3{,}000$ m), this issue must not raise the slightest doubt.

The above considerations explain to a large extent why the cases presented in studies [12, 13, 16] on the determination of curvature using the moving chord method in an in-service track were fully successful. It can be seen that the correctness of the proposed method has a strong theoretical basis.

4. Selected calculation examples

The applicability of the two proposed calculation methods for high-speed railways was demonstrated using two calculation examples. For this purpose, model geometric layouts of railway tracks created according to the principles of the analytical design method [14] were used. The individual elements of

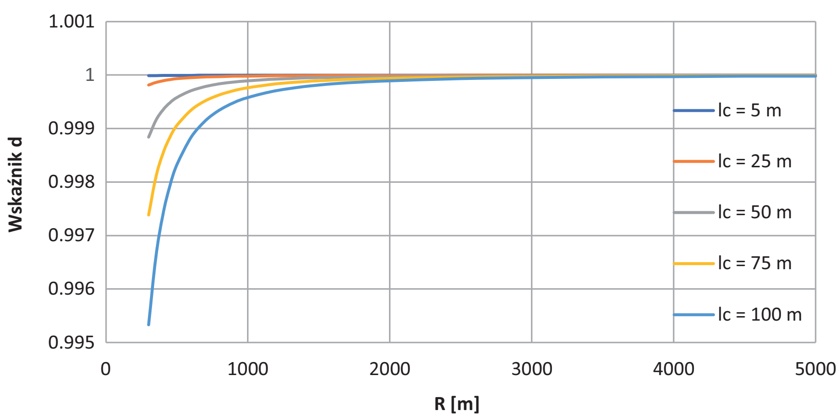

Fig. 4. Relationship graphs $d(R)$ for circular curve radii occurring on railway routes (i.e. $R = 300 \div 5000$ m) and chord length $l_c = 5 \div 100$ m [author's own elaboration]

these layouts are described by mathematical equations, which greatly facilitates the relevant analysis.

In order to maintain the applicability to reality to a greater extent, it was decided to obtain hypothetical measurement data by virtual modification of the model layouts. The railway track axis coordinates were corrected randomly at 5 m intervals, assuming a maximum error value of ±10 mm. The determination of the directional angle of the railway route and the curvature of the railway track axis was carried out for chord lengths l_c = 50 m and l_c = 100 m.

4.1. Geometric layout for the speed of 260 km/h

The first geometric layout, suitable for a speed of 260 km/h, consists of a circular curve with a radius of 5,000 m, two transition curves in the form of a Euler spiral (i.e. clothoid) with a length of 240 m and two straight sections of approximately 370 m. The total length of the layout is 3,600 m, with a curve deflection angle of α = 0.523599 rad. Its position in the PL-2000 system is shown in Figure 5. Circular curves have been marked in red, transition curves in blue and straight sections in green.

Figure 5 shows the axes of the local *x*, *y* coordinate system to which the geometric layout under consideration will be converted, according to the procedure described in [14]. The beginning of the *x*, *y* system is located at the point of intersection of the main directions of the route, while the alignment of its axis allows to obtain the symmetry of the geometric layout. The designations in Figure 5 are as follows:

- *W* – point of intersection of the main directions, the origin of the local coordinate system,
- *S* – centre of circular curve,
- P_1 – beginning of the first transition curve,
- K_1 – end of the first transition curve,
- P_2 – beginning of the second transition curve,
- K_2 – end of the second transition curve.

To perform the conversion to the local coordinate system, it is necessary to move the origin of the PL-2000 system to the point and rotate this system to the left by an angle = 0.654498 rad. As a result, the situation depicted in Figure 6 occurs.

After the virtual modification of the model geometric layout, the directional angle values were determined, assuming two chord lengths: l_c = 50 m and

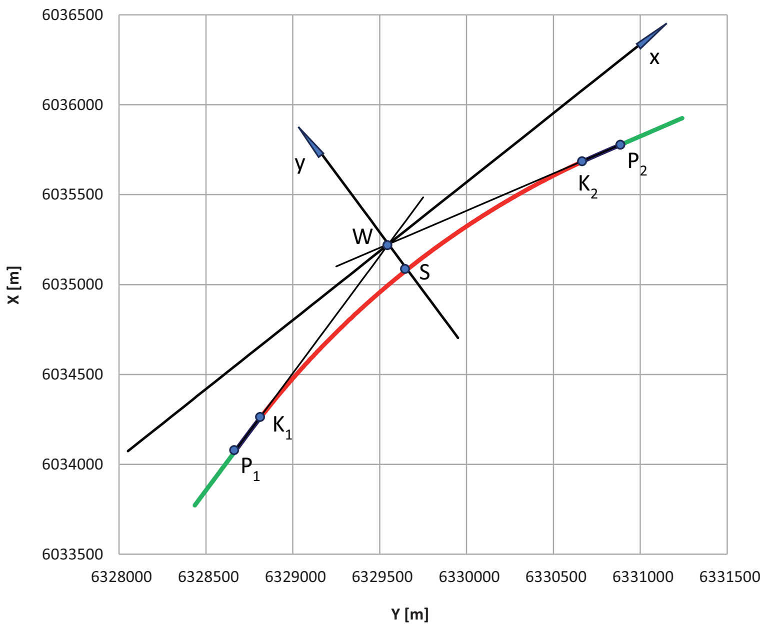

Fig. 5. Geometric layout for the speed of 260 km/h in the PL-2000 system [author's own elaboration]

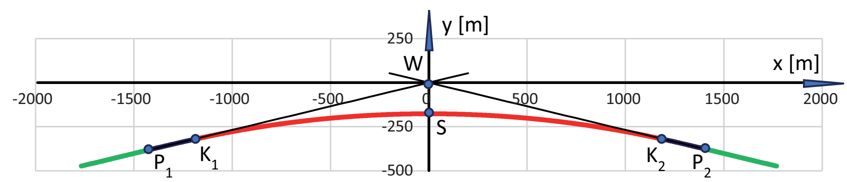

Fig. 6. Geometric layout for the speed of 260 km/h in the local coordinate system [author's own elaboration]

l_c = 100 m. The resulting Φ(*L*) graphs showed no differences between them without enlarging the scale. This is shown in Figure 7.

After enlarging the scale of the graph, certain differences in the angle Φ on the transition curves become apparent. However, these differences are small and do not exceed a value of 0.1°. This is shown in Figures 8 and 9.

As it appears, the graph of the directional angle derived from the length of the transition curve is more

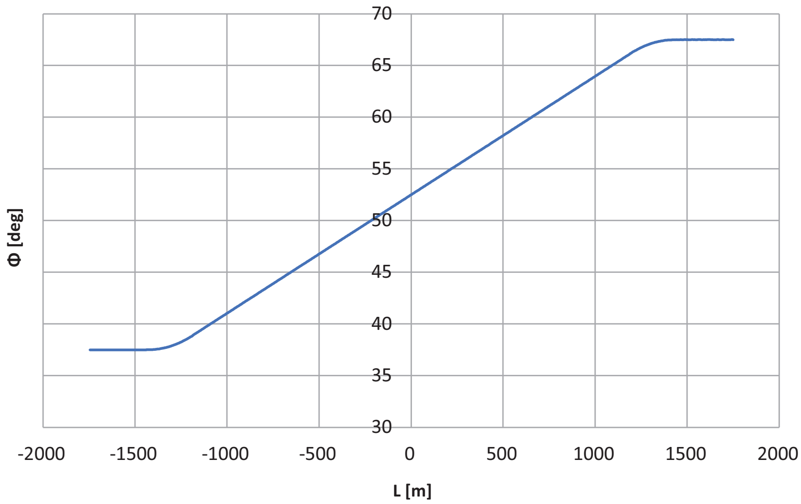

Fig. 7. Graph of the directional angle along the length of the geometric layout for a speed of 260 km/h [author's own elaboration]

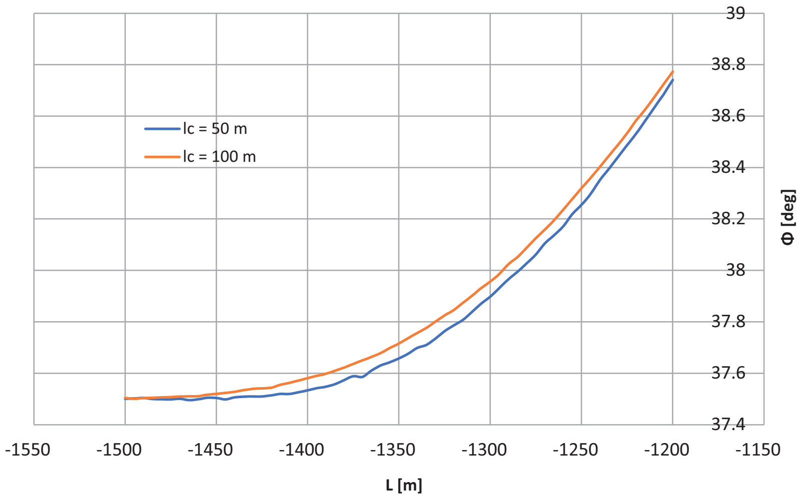

Fig. 8. Graphs of the directional angle along the length of the transition curve on the left side of the geometric layout for a speed of 260 km/h [author's own elaboration]

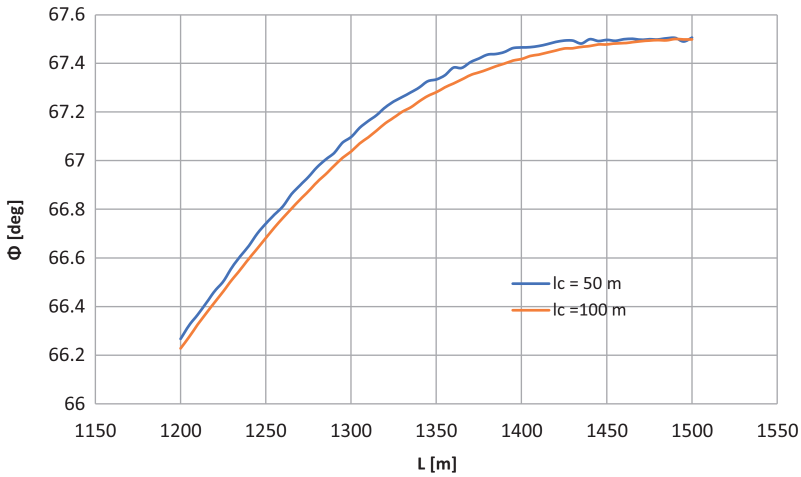

Fig. 9. Graphs of the directional angle along the length of the transition curve on the right side of the geometric layout for a speed of 260 km/h [author's own elaboration]

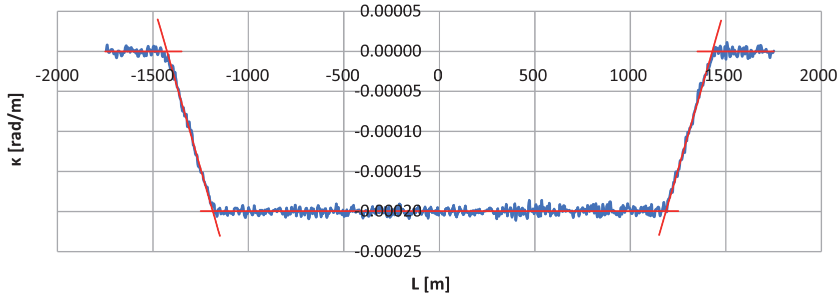

Fig. 10. Graph of curvature along the length of the geometric layout for a speed of 260 km/h determined with chord length l_c = 50 m [author's own elaboration]

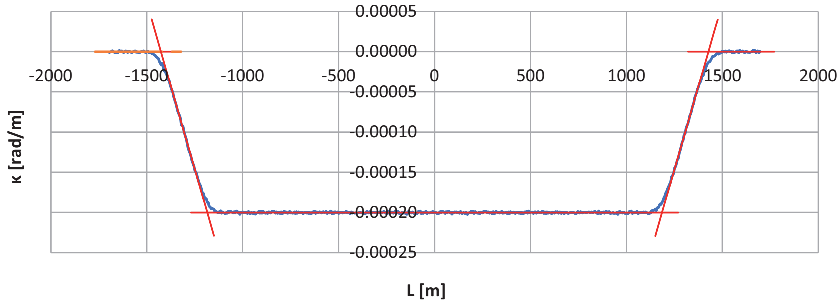

Fig. 11. Graph of curvature along the length of the geometric layout for a speed of 260 km/h determined with chord length l_c = 100 m [author's own elaboration]

regular (i.e. exhibits fewer disturbances) when using a chord length of l_c = 100 m. This observation will also be confirmed when determining the curvature of the railway track axis.

Figures 10 and 11 show graphs of the curvature of the railway track axis determined using formula (14) for = 50 m and = 100 m, respectively. The negative curvature values in these graphs are due to the shape of the track axis in Figure 6 (convexity facing upwards). The red colour indicates the regions of occurrence of each geometric element.

On straight sections the curvature is equal to zero, on a circular curve it is determined by the arithmetic mean of the curvature values at the measuring points, and on transition curves by the least-squares line. The points of intersection of the latter with the horizontal lines determine the beginnings and ends of the transition curves. Table 1 shows the values of the circular curve statistics.

The graphs in Figures 10 and 11 and the numerical values in Table 1 show the high efficiency of the moving chord method in determining the curvature of the railway track axis. As a result of the deformation of the model system, there are some disturbances in the graphs concerning curvature, but they are relatively minor, hence the practical usefulness of the $\kappa(L)$ graphs when identifying the geometric layout is unquestionable. At the same time, a clear predominance of the chord with length l_c = 100 m was revealed, discernible both visually in the drawings and based on the numerical data in Table 1. In such circumstances, this particular chord length should be recommended for use on high-speed railways.

4.2. Geometric layout for the speed of 350 km/h

The second geometric layout, suitable for a speed of 350 km/h, consists of a circular curve with a radius of 10,000 m, two transition curves in the form of a Euler spiral with a length of 280 m and two straight sections of approximately 440 m. The total length of the layout is 6,400 m, with a curve deflection angle of

Table 1

Summary values of circular curvature statistics for different chord lengths

Chord length l_c [m]	Curvature value $\bar{\kappa}$ [rad/m]	Radius of circular curve $\bar{R}$ [m]	Standard deviation σ_κ [rad/m]	Curvature index $\sigma_\kappa / \bar{\kappa}$ [%]
50	–0.0001993850	5015.421	4.35346E-06	2.183
100	–0.0002000043	4999.893	8.94415E-07	0.447

[author's own elaboration].

$\alpha = 0.523599$ rad. Its position in the PL-2000 system is shown in Figure 12. As in Figures 5 and 6, the circular curve has been marked in red, the transition curves in blue and the straight sections in green.

Figure 12 shows the axes of the local x, y coordinate system, the origin of which is located at the intersection of the main route directions. In order to obtain a symmetrical geometric layout in the local coordinate system, it is necessary to move the origin of the PL-2000 system to point W and rotate this system to the left by an angle $\beta = 2.356194$ rad. As a result, the situation depicted in Figure 13 occurs. The symbols in Figures 12 and 13 are the same as in Figures 5 and 6.

After the virtual modification of the model geometric layout, the directional angle values were determined, assuming chord length: $l_c = 100$ m. Since, in the case under consideration, the angle of inclination of the tangent $\Theta_i \in \langle 90°;180° \rangle$, formula (4) was used to determine the directional angle. The graph $\Phi(L)$ shown in Figure 14 was obtained.

Figure 15 shows the graph of the curvature of the railway track axis determined by formula (14) with

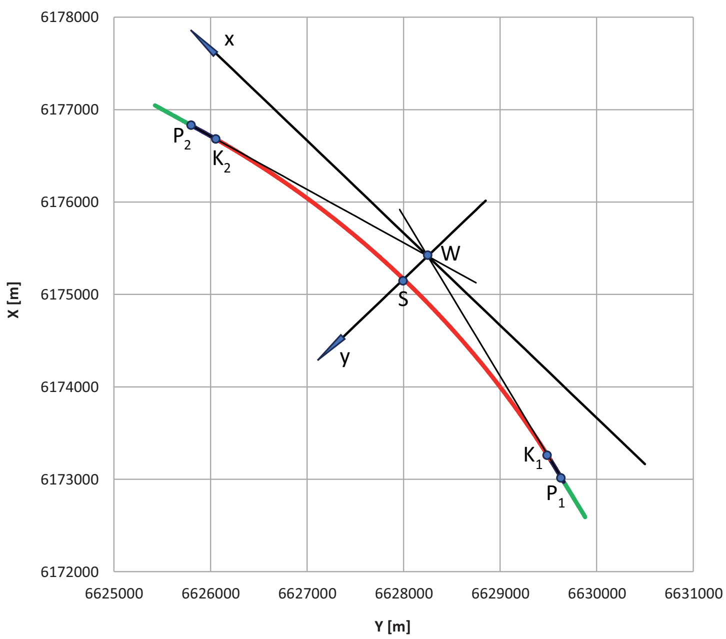

Fig. 12. Geometric layout for the speed of 350 km/h in the PL-2000 system [author's own elaboration]

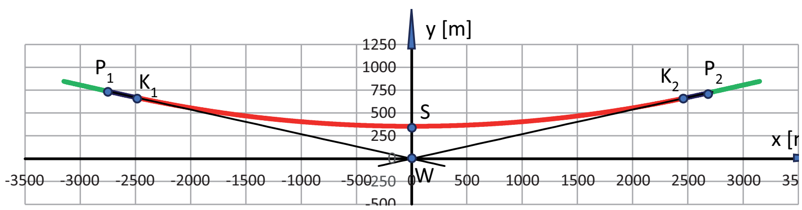

Fig. 13. Geometric system for the speed of 350 km/h in the local coordinate system [author's own elaboration]

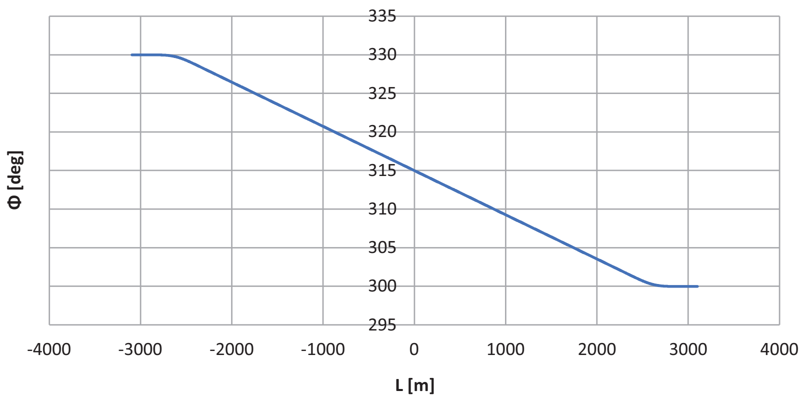

Fig. 14. Graph of the directional angle along the length of the geometric layout for a speed of 350 km/h [author's own elaboration]

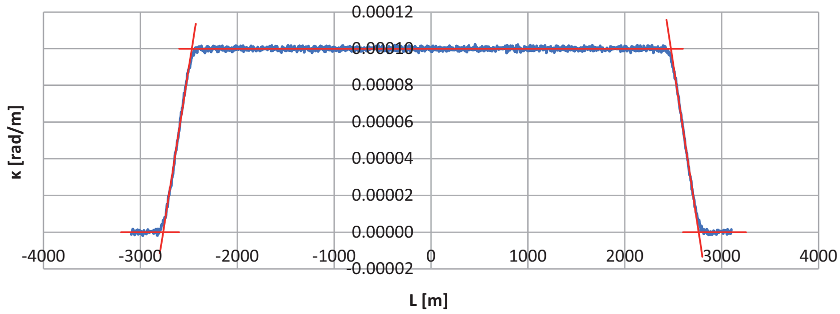

Fig. 15. Graph of curvature along the length of the geometric layout for a speed of 350 km/h determined with chord length l_c = 100 m [author's own elaboration]

a chord of length l_c = 100 m. The curvature values in this figure are positive because the geometric layout in Figure 13 has its convexity directed downward. As before (i.e. in Figures 10 and 11), the red colour indicates the regions of occurrence of each geometric element.

The values of the statistics for the circular curve are as follows:

- mean value of circular curve curvature $\bar{\kappa}$ = 0.0000999982 rad/m,
- mean value of circular curve radius $\bar{R}$ = 10000.177 m,
- standard deviation of circular curve curvature σ_κ = 9.03667E-07 rad/m,
- curvature index $\sigma_\kappa / \bar{\kappa}$ = 0.904%.

The analysis of the geometrical layout for speeds of 350 km/h has unquestionably confirmed the suitability of the moving chord method for determining the directional angle of the railway route and determining the curvature of the railway track axis on high-speed railways. A chord with a length of l_c = 100 m seems to be the best solution in this respect.

5. Conclusions

Knowledge of the current geometric layout of the railway track, obtained through measurement, allows for the elimination of any irregularities and the introduction of potential adjustments. Measurements carried out provide the basis for the identification of a railway route. It consists in determining its geometrical parameters by using the determined Cartesian coordinates of the railway track axis points in the appropriate state spatial reference system.

The current approach to the problem of identifying a measured railway route is to generate an optimised geometric layout in a given area by minimising the deviations of this layout from the measurement points (while meeting the relevant maintenance and operational requirements). As it turns out, this problem can be approached in a completely different way, as the basis for identification can be the determined curvature of the railway track axis.

Since, in each case, the identification of the geometrical layout of the railway track is done on the basis of the Cartesian coordinates of the railway track axis points (obtained during the survey), it was decided to use these coordinates in new calculation methods in which the determined inclination of the tangent to the railway track axis plays a major role. This value is determined by the identification of a virtual chord projected in the horizontal plane, which connects two points on the railway track axis (this chord is derived from a given measurement point forward and backward).

The article describes in detail the assumptions of the method for determining the directional angle of the railway route and the methods for determining the curvature of the railway track axis, together with the relevant calculation algorithms. Subsequently, an attempt was made to test the extent to which these methods correspond to the conditions specific for high-speed railways. This was shown using two calculation examples involving geometric layouts for speeds of 260 km/h and 350 km/h. In order to maintain the applicability to reality to a greater extent, it was decided to obtain hypothetical measurement data by virtual modification of the model layouts (the railway track axis coordinates were corrected randomly at 5 m intervals, assuming a maximum error value of ±10 mm). The determination of the directional angle of the railway route and the curvature of the railway track axis was carried out for chord lengths = 50 m and = 100 m.

The methods considered are related as they both employ a moving chord. When comparing the two methods, one can conclude that the method for determining the directional angle is considerably more accurate than the method for determining curvature. This is likely due to the additional simplifying assumption made when determining the curvature, concerning the distance between the tangency points of both chords to the geometric layout. However, as numerous analyses have shown, the identified inaccuracies are not significant from the perspective of the survey.

Qualitatively, the results of the analysis carried out were no different from previous analyses relating to standard railways [11–13, 16]. In particular, they confirmed beyond doubt the suitability of the methods considered for determining the directional angle of the railway route and determining the curvature of the railway track axis on high-speed railways. As shown, it would be advisable to use a chord of l_c = 100 m on these railways.

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